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Transitional Dynamics and Indeterminacy of Equilibria in an Endogenous Growth Model with a Public Input

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Abstract

The authors analyze an endogenous growth model with economy-wide increasing returns, in which a public input is essential for private production. Within such a framework, it is shown that a continuum of equilibria and global indeterminacy can arise for reasonable parameter values, simply due to the presence of endogenous public policy. This can potentially account for "leapfrogging" or retroceding growth experiences. It is demonstrated how fiscal policy can serve as a selection device among different convergent paths.

1. Introduction

The recent literature on endogenous growth attempts to address, among others, a century-old question: How do economies maintain sustained growth without bound? After many intensive explorations, it has been shown that long-run growth can be sustained if there are nondecreasing returns to reproducible factors, such as physical and human capital (Rebelo, 1991). These nondecreasing returns may arise either from external effects generated in the process of physical or human capital accumulation, as, for example, in Romer (1986) and Lucas (1988), or from the presence of productive public services (Barro, 1990; Barro and Sala-i-Martin, 1992; Corsetti and Roubini, 1996). Methodologically, one noticeable contribution of this literature is the reconciliation between increasing returns at the aggregate level and competitive behavior by firms.

Lately, significant progress has been made on the transitional dynamics of endogenous growth models. In particular, it has been realized that the introduction of external effects may lead to indeterminacy of equilibria; i.e., there may exist multiple equilibrium paths associated with given initial conditions (Benhabib and Perli, 1994; Boldrin and Rostichini, 1994; Xie, 1994; Bond et al., 1996).[1] Furthermore, as shown in these studies, indeterminacy is not only a theoretical possibility, but it may occur well within the range of parameter values that are empirically realistic and plausible. From an empirical perspective, the existence of indeterminacy in endogenous growth models can potentially explain an important question posed by, among others, Lucas (1993): Why would two different countries, such as South Korea and the Philippines, whose initial conditions were so close, differ so much in their later growth performance? In the presence of indeterminacy, it is possible that two identically endowed countries choose different convergent paths; i.e., they consume, invest and allocate productive factors differently.

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Given these findings, the following question, which is the purpose of this paper, arises: Can public policy act as a selection device among different convergent paths? More specifically, this paper first shows that agents' expectations about government policy may be self-fulfilling. Thus, simply the presence of endogenous public policy may generate multiple equilibria with global indeterminacy. Second, it demonstrates how the government may change this and use public policy as a selection device among different Pareto-rankable growth paths. These findings can explain leapfrogging and retroceding growth experiences as a result of differences in the way public policy is conducted.[2]

Section 2 of the paper outlines the theoretical framework. To avoid unnecessary complications and to highlight the basic forces at work, we adopt a simple one-sector endogenous growth model that introduces explicitly public policy. A natural candidate is the Barro (1990) model that studies the link between government spending and economic growth. As is well known, however, the original Barro model does not exhibit any transitional dynamics and hence, as it stands, it is not suitable for the study of the indeterminacy issue, which is the main focus of this paper. To rectify this lacuna, we generalize the model by introducing an endogenous labor supply decision, along the lines suggested by Benhabib and Farmer (1994). With this modification, not only can transitional dynamics and indeterminacy occur but also multiple balanced growth equilibrium paths may emerge as well.

The type of government spending that we consider is an indispensable input to private production, which is best interpreted as spending on public infrastructure, such as transportation, communication, sewers, and water systems. It is well known that infrastructure expenditure is particularly important in the growth process especially for less-developed countries. For example, Easterly and Levine (1997) and Collier and Gunning (1999) find that inadequate infrastructure is among the main variables that account for the slow growth found among countries of sub-Saharan Africa.[3]

Section 3 presents the balanced growth equilibrium analysis of the model, while section 4 examines the transitional dynamics and demonstrates the possibility of indeterminacy and multiplicity of equilibria. The intuition is as follows. Each agent does not know the path of public services, which in effect depends, through the government budget constraint, on the decisions made by all other agents. Thus, if agents expect a high level of productive public services then they will increase their, otherwise low, demand and supply of capital and labor. The subsequent increase in the level of these inputs will generate higher incomes, which lead, in turn, to higher tax revenue (through an existing proportional income tax) and hence to a higher level of public services. It is in this way that agents' expectations regarding future government policy may be self-fulfilling. We derive conditions that give rise to multiple equilibria and global indeterminacy.

Section 5 demonstrates that, by pre-committing to the appropriate level of public services, the government can coordinate agents' decisions and lead the economy to any *equilibrium* it desires. Put differently, fiscal policy can serve as a selection device among different equilibrium paths. Section 6 concludes the paper.

2. The Economy

Consider an economy which consists of a large number of infinite-lived identical consumers and a large number of identical firms.[4] To simplify the exposition, we assume that the number of firms equals the number of consumers. Furthermore, time t is continuous, starting at $t = 0$, and all markets are assumed to be competitive.[5]

Each consumer seeks to maximize her lifetime utility, V, given by

$$V = \int_0^\infty \left\{ \ln c(t) - [u(t)]^{1-\varepsilon} \big/ (1-\varepsilon) \right\} \exp(-\rho t) dt, \tag{1}$$

subject to the budget constraint

$$\dot{k}(t) = (1-\tau)[w(t)u(t) + r(t)k(t)] - c(t), \tag{2}$$

and the initial value of her wealth

$$k(0) = k_0 > 0, \tag{3}$$

where c, u, k, w, r, and τ denote, respectively, per-capita consumption, work effort, per-capita capital, the wage rate, the rental rate of capital, and a flat income tax rate. The two preference parameters, ρ, the rate of time preference, and ε, the elasticity of marginal disutility of work, satisfy the following constraints: $\rho > 0$ and $\varepsilon \leq 0$. Finally, throughout the paper, $\dot{z} \equiv dz/dt$ denotes the time derivative of any variable z.

Each firm has access to a production technology described by[6]

$$y = Ak^\alpha g^{1-\alpha} u^{1-\alpha}, \tag{4}$$

where y and g are per-capita output and government spending, respectively, A (>0) denotes simply a scale parameter, and $\alpha \in (0, 1)$.

Before we proceed, a justification regarding the functional forms is in order. With regard to preferences (equation (1)) we offer the following arguments. First, per-capita consumption enters the instantaneous utility function log-linearly because, in the presence of separability between consumption and labor and of a Cobb–Douglas production function, it is the only formulation consistent with stationary labor supply in a growing economy (King and Rebelo, 1988; Benhabib and Farmer, 1994). Second, the disutility of labor is modeled as a power function because it allows us to derive an analytical expression for the labor supply function. Nevertheless, as we demonstrate in the Appendix, using the more general functional form, $\ln c + v(u)$, where $v' < 0$ and $v'' < 0$, does not alter our results. Finally, recent work by Bennett and Farmer (1998) suggests that our results can be extended to the following case of nonseparable utility: $\{[cv(u)]^{1-\sigma} - 1\}/(1 - \sigma)$.

Consider next the production specified in equation (4). First, it may be noticed that we assume constant returns to scale in k and g, holding u fixed, as well as constant returns to scale in k and u, holding g fixed. The former is necessary *only* for the existence of perpetual growth, as in Barro (1990); i.e., it is not necessary for the presence of indeterminacy. The latter is required for the model to be consistent with the assumption of perfect competition (with constant returns to scale in k and u alone, if each factor is paid its marginal product then, by Euler's theorem, the value of the privately produced output is exhausted by the payments to the private inputs and thus $y = wu + rk$). Second, the government services, g, are nonexcludable and rival. Furthermore, they are taken as parametrically given by each individual firm. Third, since there are constant returns in k and u, simply the presence of g implies that there are increasing returns in all three inputs, k, u, and g. Crucially, however, for the presence of price-taking behavior and the existence of competitive prices these are *external* increasing returns that manifest themselves at the aggregate level. In other words, there are increasing returns at the level of the economy (i.e., with respect to all inputs), and constant returns at the level of the firm (i.e., with regard to the inputs, k and u, that are controlled by the firm).[7] Finally, we note that to be able to maintain tractability and to solve analytically for the transitional path of the economy, a Cobb–Douglas specification is adopted.

The government operates in the following way. At each instant ("period") it raises revenue, $\tau(wu + rk)$, by imposing a proportional income tax at the rate τ. All revenue raised is then used to finance public services, g. Since output is exhausted (recall that the production function exhibits constant returns to scale in the two private inputs, u and k), the instantaneous government budget constraint can be written as[8]

$$g = \tau y = \tau A k^{\alpha} g^{1-\alpha} u^{1-\alpha}. \tag{5}$$

It should be noted that, different from Barro (1990), the model allows for an endogenous labor supply, a feature often encountered in models involving indeterminacy. In contrast, however, to other models which result in global indeterminacy, public spending is an indispensable input in the production process, allowing thus for an explicit link between government spending and economic growth.

3. Equilibrium Analysis

To derive the equilibrium path(s) of this economy, we first solve the optimization problems of the representative consumer and of the representative firm. We then combine the first-order necessary conditions from each problem with the government budget constraint and the market-clearing conditions.

The representative consumer maximizes (1) subject to (2) and (3), taking the tax rate, τ, and the paths of $w(t)$ and $r(t)$ as given. Consider next the current-value Hamiltonian for this system

$$H = \ln c - (u)^{1-\varepsilon}/(1-\varepsilon) + \lambda[(1-\tau)(wu + rk) - c],$$

where λ denotes the costate variable associated with the budget constraint (2). The first-order necessary conditions are

$$c^{-1} = \lambda, \tag{6}$$

$$u^{-\varepsilon} = \lambda(1-\tau)w, \tag{7}$$

$$\dot{\lambda} = \rho\lambda - \lambda(1-\tau)r, \tag{8}$$

$$\lim_{t\to\infty} \lambda(t)k(t)\exp(-\rho t) = 0, \tag{9}$$

and equations (2) and (3). Moreover, since the instantaneous utility function is concave in c, k, and u, and the opportunity set is a closed and convex region, these first-order necessary conditions are also sufficient for optimality (recall that $w(t)$ and $r(t)$ are considered as given functions of time by each agent).[9]

Each firm, taking the level of government spending as given, attempts to maximize its profits, by employing labor and capital up to the point where the marginal product of each factor equals its market price. Thus

$$w = (1-\alpha)A k^{\alpha} g^{1-\alpha} u^{-\alpha}, \tag{10}$$

$$r = \alpha A k^{\alpha-1} g^{1-\alpha} u^{1-\alpha}. \tag{11}$$

It should be noted that, in every "period" t, the optimal program of each consumer, $\{c(t), u(t), k(t)\}$, and of each firm, $\{k(t), u(t)\}$, will depend (parametrically), among other things, on the level of public services, $g(t)$.[10]

Next, rewrite the government budget constraint, equation (5), as

$$g = (\tau A)^{1/\alpha} k u^{(1-\alpha)/\alpha}. \tag{12}$$

One can then combine (12) with the first-order necessary conditions (2), (6)–(8), (10), and (11) to eliminate g. More specifically, substituting (10)–(12) in (2) yields

$$\dot{k} = (1-\tau)A^{1/\alpha}\tau^{(1-\alpha)/\alpha}u^{(1-\alpha)/\alpha}k - c. \tag{13}$$

Furthermore, substituting (6), (10), and (12) in (7), we have

$$u^{1-\varepsilon-(1-\alpha)/\alpha} = (1-\alpha)(1-\tau)A^{1/\alpha}\tau^{(1-\alpha)/\alpha}\,k/c. \tag{14}$$

Finally, combining (6), (8), (11), and (12) yields

$$\dot{c}/c = \alpha(1-\tau)A^{1/\alpha}\tau^{(1-\alpha)/\alpha}u^{(1-\alpha)/\alpha} - \rho. \tag{15}$$

Equations (13)–(15), together with the initial condition, equation (3), and the transversality condition, equation (9), rewritten as $\lim_{t\to\infty}[k(t)/c(t)]\exp(-\rho t) = 0$, characterize the equilibrium path(s) of the economy. Once the equilibrium paths of $c(t)$, $k(t)$, and $u(t)$ are determined, we can easily obtain the paths of all other variables by substituting in the appropriate equations.

Consider next the definition of an equilibrium balanced growth path.

DEFINITION 1. *Consider a path $\{c(t), k(t), u(t)\}_{t=0}^{\infty}$ that satisfies the system of equations (13)–(15). We call it an (equilibrium) balanced growth path (BGP) if the growth rates of c and k, $\dot{c}/c$ and $\dot{k}/k$, are constant over time and $\dot{u} = 0$.*

Using equations (12) and (13)–(15), it is straightforward to show that, along a BGP, consumption, capital stock and government spending (in per-capita terms) grow at the same rate (θ); that is

$$\dot{c}/c = \dot{k}/k = \dot{g}/g = \theta.$$

In addition, equations (4), (6), (10), and (11) imply

$$\dot{y}/y = \dot{w}/w = -\dot{\lambda}/\lambda = \theta \text{ and } \dot{r} = 0.$$

Notice, from equations (13)–(15) that if the amount of work effort, u, is exogenously given, as in Barro (1990), then $\dot{c}/c$ and $\dot{k}/k$ will be constant at every instant and hence the economy will always be on a BGP; i.e., there will be no transitional dynamics.

4. Multiple Equilibria and Indeterminacy

Since one of our objectives is to study the transitional dynamics of this economy, we next reduce the dimension of the system to a more tractable one. Define first $x(t) \equiv c(t)/k(t)$, $\forall t \geq 0$. Equation (14) can then be written as

$$u = \left\{(1-\alpha)(1-\tau)A^{1/\alpha}\tau^{(1-\alpha)/\alpha}\right\}^{\frac{1}{1-\varepsilon-(1-\alpha)/\alpha}} x^{\frac{1}{(1-\alpha)/\alpha-(1-\varepsilon)}}. \tag{16}$$

Combining (16) with (13) and (15) yields the following law of motion for $x(t)$:

$$\dot{x}/x = (\dot{c}/c) - (\dot{k}/k) = x - D^{1-\delta}x^{\delta} - \rho \equiv h(x), \tag{17}$$

where $D \equiv (1-\alpha)(1-\tau)A^{1/\alpha}\tau^{(1-\alpha)/\alpha}$ and $\delta \equiv (1-\alpha)/[1-\alpha-\alpha(1-\varepsilon)]$. Since $\alpha \in (0, 1)$ and $\varepsilon \leq 0$, it follows that $\delta \in (-\infty, 0) \cup (1, \infty)$. Next, using the definition of $x(t)$, we rewrite equation (9) as

$$\lim_{t\to\infty}[1/x(t)]\exp(-\rho t) = 0. \tag{18}$$

Equation (17) then, with the initial condition (3) and the transversality condition (18), forms a first-order nonlinear system that describes the global dynamics of the economy. A few remarks regarding this system are in order. First, for obvious reasons, we restrict our attention to the case where x takes values in the set of positive real numbers, R_{++}. Second, notice that equation (17) does not generate closed orbits in the $\dot{x} - x$ phase-plane and hence it does not accept any periodic solutions. It follows then that, if $u(t)$ and hence $x(t)$ assume interior values, all solutions, $x(t)$ for $t \geq 0$, of (17) converge to a steady-state equilibrium point x^*, such that $\dot{x} = x^*h(x^*) = 0$. Third, each steady-state equilibrium point of (17) corresponds to a BGP in the system of (13)–(15), along which $\dot{x}/x = (\dot{c}/c) - (\dot{k}/k) = 0$. Finally, any such solution will satisfy the transversality condition (18), since $k(t)$ and $c(t)$ will eventually grow at the same rate, implying that $x(t) = c(t)/k(t)$ will eventually assume a constant value.

The next definition introduces the concept of *global indeterminacy*.

DEFINITION 2. *We say that global indeterminacy occurs if given an initial condition* $k(0)$*, there exist at least two initial conditions* $x_i(0) \in R_{++}$, $i = 1, 2$*, such that the solutions* $x(x_i(0), t)$ *converge to a steady-state equilibrium point. In this case, we also say that all steady-state equilibria (and the associated BGPs) are globally indeterminate. Finally, steady-state equilibrium points (and the associated BGPs) that are not globally indeterminate are said to be globally determinate.*

Since the initial value of $c(0)$—and hence of $x(0) \equiv c(0)/k(0)$—is not predetermined, it follows from Definition 2 that if there are multiple equilibria then global indeterminacy occurs, regardless of the stability properties of these equilibria. Put differently, the multiplicity of steady-state equilibria in this economy is a sufficient condition for global indeterminacy. We further distinguish between the following two types of steady states and of BGPs.

DEFINITION 3. *A steady-state equilibrium point* x^* *of equation (17) and the BGP associated with it are said to be locally indeterminate if there exists a nondegenerate interval* $\Omega \subseteq R_{++}$ *such that for every initial condition* $x(0) \in \Omega$ *the solution* $x(x(0), t)$ *converges to* x^**. Steady states (and the associated BGPs) that are not locally indeterminate are said to be locally determinate.*

The following statements can then easily be proved using Definitions 2 and 3. First, a globally determinate steady-state equilibrium is also locally determinate. Second, a locally indeterminate steady-state equilibrium is also globally indeterminate. (The converse of each of these two statements, however, is not always true.) Third, every unstable steady-state equilibrium is locally determinate (if it is unique then it is also globally determinate). Finally, every stable steady-state equilibrium is locally and, by virtue of the second statement, globally indeterminate. Notice, in particular, that in the case of a stable-steady state, there exists a continuum of initial values $x(0) \equiv c(0)/k_0$, indexed by $c(0)$, which lead to the stable point. Consider then the following proposition (proved in the Appendix).

PROPOSITION 1.

(i) *If* $\delta \in (-\infty, 0)$*, then there exists a unique but unstable BGP.*
(ii) *If* $\delta \in (1, \infty)$*, then there exist at most two BGPs. If there are exactly two, one is unstable and determinate and the other is asymptotically stable and indeterminate.*

Figures 1 and 2 provide a graphical illustration of this proposition. Figure 1 depicts case (i) where there is only one unstable and hence globally determinate steady-state

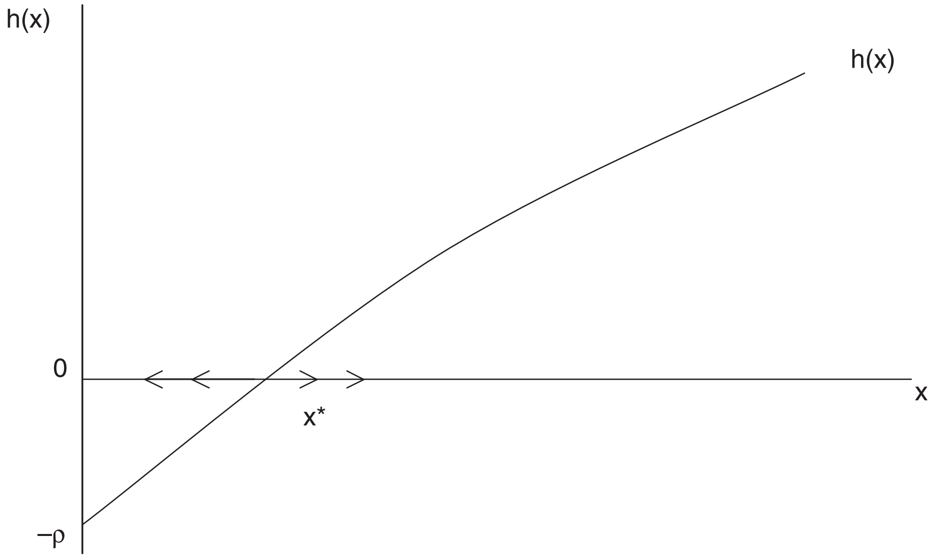

Figure 1. A Unique and Determinate BGP; δ ∈ (−∞, 0)

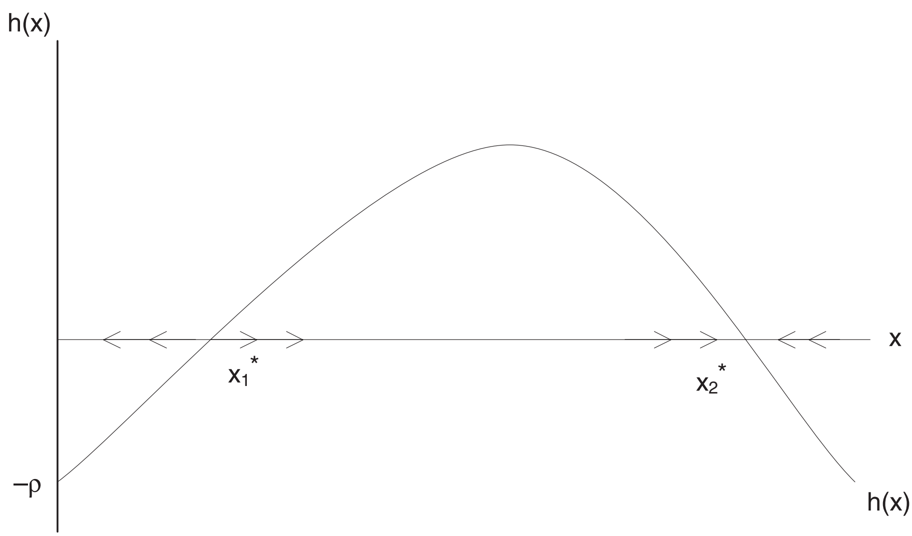

Figure 2. Two BGPs; δ ∈ (1, ∞)

equilibrium, while Figure 2 presents the case where there are two steady-state equilibria. Point x_1^* represents a locally unstable and determinate equilibrium. Point x_2^*, on the other hand, is a locally stable and hence indeterminate equilibrium; given k_0, any value of $c(0)$ such that $c(0)/k_0 > x_1^*$ gives rise to an equilibrium trajectory that converges to x_2^*. Intuitively, the multiplicity of equilibria occurs because if firms (consumers) anticipate a high level of public services then they will demand (supply) more capital and labor. The subsequent expansion in the values of k and u generates higher incomes, higher tax revenue, and hence a higher level of public services. An equilibrium with a low level of public services is also possible for similar reasons. Thus, agents' expectations regarding the future level of public services are self-fulfilling.

A question arises next as to whether the case of an indeterminate equilibrium is empirically relevant. Recall that, in the present model, indeterminacy can occur if $\delta \in (1, \infty)$, or, equivalently, if $1 - \varepsilon < (1 - \alpha)/\alpha$. It is obvious that there exists a wide range of plausible parameter values for which indeterminacy can occur. To see the economic explanation of the condition $1 - \varepsilon < (1 - \alpha)/\alpha$, derive first the labor supply elasticity, $\partial(\ln u)/\partial(\ln w) = -1/\varepsilon$, using equation (7). Next, derive the elasticity of labor demand $\partial(\ln u)/\partial(\ln w) = \alpha/(1 - 2\alpha)$, using equations (10) and (12). The necessary condition for indeterminacy then requires that the elasticity of labor supply is greater than the elasticity of labor demand.[11] Farmer and Guo (1995) provide strong empirical support for this condition.[12] Furthermore, as noted above, other papers in which indeterminacy arises suggest that our main argument is valid even in more general settings; for example when one considers nonseparable preferences, as in Bennett and Farmer (1998), or allows for more than one sector in the economy, as in Benhabib and Farmer (1996). In these cases, however, the necessary and sufficient conditions for indeterminacy become far more plausible. For example in the case where there exist more than one sectors in the economy, instead of the aforementioned relationship between the elasticities of labor supply and labor demand, one simply requires a degree of increasing returns as mild as 1.07 (Benhabib and Farmer, 1996, 1999).

Given these findings, it is important to examine whether government policy can be used as a selection device among these different convergent paths. The next section examines more closely the case of indeterminacy and suggests policies that can enhance economic growth and promote welfare.

5. Policy Implications: Fiscal Policy as a Selection Device

The fundamental reason behind the possibility of multiple equilibria and indeterminacy lies in the government budget constraint; more specifically, it is the fact that agents do not realize that an increase in their levels of capital and work effort will lead to a higher level of public services. In this section, we show that the government can select any equilibrium path it desires, out of the infinite number that exist in this economy. Importantly, we assume that the government has the ability to commit to a certain path of government spending.

For simplicity, suppose the government desires the economy to jump on the BGP associated with x_2^* from the date $t = 0$ onwards. This requires (from equation (16)) that for every $t \geq 0$

$$u(t) = u_2^* = \left\{(1-\alpha)(1-\tau)A^{1/\alpha}\tau^{(1-\alpha)/\alpha}\right\}^{\frac{1}{1-\varepsilon-(1-\alpha)/\alpha}}(x_2^*)^{\frac{1}{(1-\alpha)/\alpha-(1-\varepsilon)}}.$$

Equation (12) then implies that the government should pre-commit to a path of public services, $g_2^*(t) = (\tau A)^{1/\alpha}(u_2^*)^{(1-\alpha)/\alpha}k(t)$. To be more specific, consider the following example which reverses the order of the above exercise.

Example

Assume that $\alpha = 1/3$, $A = 2.125$, $\varepsilon = 0$, $\rho = 0.03$, $\tau = 0.25$, $k_0 = 1$. Solving the system as before in terms of $q(t) \equiv g(t)/k(t)$, we obtain the following differential equation:

$$\dot{q}(t)/q(t) = [(1-\alpha)/(1-2\alpha)]\left\{(1-\alpha)(1-\tau)\tau^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}q(t)^{\frac{1-2\alpha}{1-\alpha}} - (1-\alpha)(1-\tau)q(t)/\tau - \rho\right\}$$

or, by substituting the parameter values specified above:

$$\dot{q}(t)/q(t) = 2\{0.774q(t)^{0.5} - 2q(t) - 0.03\}. \tag{19}$$

Suppose that, in the beginning of "period" $t = 0$, the government pre-commits to a level of public services, $g(0) = 0.118$, so that $q(0) \equiv g(0)/k_0 = 0.118$. According to equation (19), the economy then will jump immediately to the stationary point, $q_2^* = 0.118$, which corresponds to an equilibrium BGP path along which $x(t) \equiv c(t)/k(t) = 0.266$, $u(t) = 0.887$, $q(t) \equiv g(t)/k(t) = 0.118$, $\theta(t) = 0.088$, for every $t \geq 0$, and results in a level of lifetime utility $V = 57.431$. If, on the other hand, the government announces $g(0) = 0.00191$, then $x(t) \equiv c(t)/k(t) = 0.0338$, $u(t) = 0.113$, $q(t) \equiv g(t)/k(t) = 0.00191$, $\theta(t) = -0.0281$, for every $t \geq 0$, and $V = -114.550$. In a similar manner, the government can direct the economy to any of the other equilibrium paths which eventually will lead to the high growth equilibrium, resolving thus not only the global but also the local indeterminacy.

Once again, the multiplicity and indeterminacy of equilibria occur because each agent does not know the path of public services, which in effect depends, through the government budget constraint, on the decisions made by all other agents. By precommitting to a particular level of public services the government resolves the ambiguity and coordinates all these decisions. Finally, consider the following proposition (proved in the Appendix).

Proposition 2. *Suppose one allows the tax rate, τ, to vary over time. Consider the problem of finding the optimal path from the standpoint of a benevolent government, which takes into account the behavior of the representative agent and the representative firm. Then the optimal tax rate is constant and equal to $1 - \alpha$, $\forall t$.*

Proposition 2 has important implications. For example, a benevolent government always sets the optimal tax rate at the constant level $1 - \alpha$ (the elasticity of output with respect to g), which is independent of time. Even if the government is given the opportunity to re-optimize at a later point and change τ, it will do so. In other words, *given our framework*, this policy is not only optimal but also credible (time-consistent).

6. Conclusions

By analyzing the implications of an endogenous growth model with a public input, we have shown that simply the presence of endogenous public policy may give rise to multiple growth paths with global indeterminacy. This is not only theoretically possible, but also empirically relevant. With regard to policy implications, we have demonstrated how fiscal policy can serve as a potential tool which can improve growth performance and enhance welfare. Although our analytical framework is fairly simple, our findings are somewhat robust with respect to perturbations in functional forms and parameter values.

We believe that our findings are important in explaining diverse growth experiences. Nevertheless, whether public policy can always serve as a selection device among different equilibria, or whether one has to rely on certain political, cultural, and institutional features as well, remains an open question. Furthermore, if differences in growth performance are attributed to differences in public policy, one wonders why public policy differs so much across countries. The answer may hinge on two important assumptions made in this paper, which are both related to the issue of government credibility. First, to act as a coordinator, the government must be able to commit to a certain action. Recall that, in discussing the role of fiscal policy as a selection device, we assumed that the government was able to move first and commit to a certain path of government spending. If this is not so, then the government policy is not credible (time-consistent). Second, we also assumed that the government shares the same interests with the private agents. For example, in deriving the optimal policy, we assumed that the government's objective is to maximize the representative agent's lifetime utility. If this assumption is violated then, once again, the optimal government policy will not be credible. The issue of government credibility may be of real value in considering the historical differences in coordination performance of countries with similar conditions at some prior point in time.[13] Perhaps, the literature identified as "political economy" that studies these issues (e.g., Persson and Tabellini, 1999) can provide a useful starting point to address these questions. There can be no doubt that further research is needed in this area.

Appendix

A More General Functional Form

We demonstrate that using the more general utility function $\ln c + v(u)$, where $v' < 0$ and $v'' < 0$, does not alter our results. Setting up the Hamiltonian and differentiating yields the same first-order conditions with the exception of (7), which now becomes

$$-v'(u) = \lambda(1-\tau)w. \tag{A1}$$

Furthermore, equation (13) changes to

$$-v'(u)u^{(2\alpha-1)/\alpha} = (1-\alpha)(1-\tau)A^{1/\alpha}\tau^{(1-\alpha)/\alpha}\,k/c. \tag{A2}$$

The dynamic system then that describes the equilibrium path(s) consists of (13), (15), and (A2), together with (3) and the transversality condition. Nevertheless, given the general form of $v(u)$, one cannot derive an explicit solution of u in terms of x, as in equation (16). Instead, we can rewrite (A2) as

$$-v'(u)u^{(2\alpha-1)/\alpha} = Dx^{-1}, \tag{A3}$$

where it may be recalled that $D \equiv (1-\alpha)(1-\tau)A^{1/\alpha}\tau^{(1-\alpha)/\alpha}$ and $x(t) \equiv c(t)/k(t)$. Equation (A3) defines implicitly a function ϕ, where $u = \phi(x)$. Thus, combining (13) and (15) yields

$$\dot{x}/x = x - D(\phi(x))^{(1-\alpha)/\alpha} - \rho \equiv h(x), \tag{A4}$$

which replaces (17). If we let $\varepsilon(u) \equiv -v''u/v'$ then our results regarding the presence of indeterminacy are similar to those presented in Proposition 1. In particular, notice that the necessary condition for indeterminacy $\delta(u) \in (1, \infty)$, or $1 - 2\alpha + \alpha\varepsilon(u) > 0$, is less restrictive, since it is required to hold only in the neighborhood of a *BGP*.

Proof of Proposition 1

(i) Consider first the case where $\delta \in (-\infty, 0)$. Trivially the function $h(x) = x - D^{1-\delta}x^{\delta} - \rho$ is continuous in R_{++}. Furthermore, $\lim_{x\to 0}h(x) = -\rho$, $\lim_{x\to\infty}h(x) = \infty$, $h'(x) = 1 - \delta D^{1-\delta}x^{\delta-1} > 0$. Thus, there exists a unique point x^*, such that $h(x^*) = 0$, and hence a unique BGP such that $x(t) = x^*$ for every $x(t)$ along this path. Since $h'(x) > 0$, it follows that $\dot{x}(t) < 0$ for $x(t) < x^*$ and $\dot{x}(t) > 0$ for $x(t) > x^*$; that is, x^* is unstable. This implies that there exists only one initial value, $x(0) = x^*$, such that a trajectory that starts from $x(0)$ (trivially) converges to x^*; hence the unique BGP is determinate (see Figure 1).

(ii) If $\delta \in (1, \infty)$, the function $h(x)$ need no longer be monotone. More specifically, $h(x)$ has the following properties: $\lim_{x\to 0}h(x) = -\rho$, $\lim_{x\to\infty}h(x) = -\infty$, $\lim_{x\to 0}h'(x) = 1$, $\lim_{x\to\infty}h'(x) = -\infty$, $h''(x) = -\delta(\delta - 1)D^{1-\delta}x^{\delta-2} < 0$. Since a concave curve can intersect the horizontal axis at most twice, there exist at most two BGPs. Suppose next that there are indeed two intersections at points x_1^* and x_2^* where $x_1^* < x_2^*$. Given the properties of $h(x)$, it follows that it intersects the horizontal axis from below at x_1^* and from above at x_2^*. Hence, x_1^* is unstable and x_2^* is asymptotically stable (see Figure 2). The BGP associated with x_1^* resembles the one examined in case (i) and is (locally) determinate. The stationary point x_2^* on the other hand, is indeterminate since there exists a continuum of initial conditions, namely every $x(0) > x_1^*$, and hence a continuum of equilibrium paths that converge to x_2^*. □

Proof of Proposition 2 (Optimal Policy)

We sketch the properties of the optimal path from the standpoint of a benevolent government, which takes into account the behavior of the representative agent and the representative firm. Specifically, the government is assumed to maximize the representative agent's lifetime utility (1), subject to the constraints for the agent's optimization problem, (2)–(3), the existing production technology (4), the government budget constraint (5), and the first-order necessary conditions to the agent's and the firm's problems, (6)–(11). It should be noted that the control variables in this problem are c, u, g, y, τ, w, and r and the stock variables are k and λ. Using equations (4)–(7) and (10)–(11) to substitute away c, u, g, y, w, and r, the problem simplifies to:[14]

$$\max\int_0^\infty \left\{ -\ln\lambda(t) - \left[B(1-\tau(t))\tau(t)^{\frac{1-\alpha}{\alpha}}\lambda(t)k(t)\right]^{\frac{\alpha}{1-\alpha}(1-\varepsilon)\delta} \Big/ (1-\varepsilon)\right\}\exp(-\rho t)dt \qquad \text{(A5)}$$

s.t.

$$\dot{k}(t) = [1/(1-\alpha)]B^{1+\delta}(1-\tau(t))^{1+\delta}\tau(t)^{(1-\alpha)(1+\delta)/\alpha}\lambda(t)^{\delta}k(t)^{1+\delta} - \lambda(t)^{-1} \qquad \text{(A6)}$$

$$\dot{\lambda}(t) = \rho\lambda(t) - [\alpha/(1-\alpha)]B^{1+\delta}(1-\tau(t))^{1+\delta}\tau(t)^{(1-\alpha)(1+\delta)/\alpha}\lambda(t)^{1+\delta}k(t)^{\delta}, \qquad \text{(A7)}$$

where $B \equiv (1 - \alpha)A^{1/\alpha}$. Letting H_1 denote the current-value Hamiltonian for this problem and ϕ_k and ϕ_λ stand for the costate variables associated with (A6) and (A7), one finds that the optimal path is described by $\tau = 1 - \alpha$, $\forall t$, $\dot{\phi}_k(t) = \rho\phi_k(t) - (\partial H_1(t)/\partial k(t))$, $\dot{\phi}_\lambda(t) = \rho\phi_\lambda(t) - (\partial H_1(t)/\partial\lambda(t))$, (A6), (A7), $(\partial H_1(0)/\partial\lambda(0)) = 0$, and $\phi_\lambda(0) = 0$, where the last two equations follow from the fact that the initial value of λ is unconstrained. Notice, in particular, that the optimal path of τ, and thus of g/y, remains constant over time and equal to $1 - \alpha$, the elasticity of output with respect to g.

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Notes

1. Indeterminacy of equilibria can arise also in the one-sector neoclassical growth model, in the presence of input externalities (Benhabib and Farmer, 1994) or endogenous fertility choice (Palivos, 1995).
2. In a recent paper, Brezis et al. (1993) study the cycles in national leadership. They suggest a mechanism that can explain patterns of leapfrogging as a result of a different response to occasional major changes in technology. Our paper suggests an alternative explanation.
3. Other studies that document the importance of infrastructure in the growth and development process include Aschauer (1989) and Easterly and Rebelo (1993).
4. Our infinite-horizon representative agent framework can be justified by appealing to the work of Barro (1974). Nevertheless, see Kirman (1992) for a critical discussion of some of the aggregation issues involved. See also Balasko et al. (1995) for a recent overview of the issue of indeterminacy in overlapping generations models.
5. See Benhabib and Farmer (1994) for the possibility of indeterminate equilibria in a monopolistically competitive model.
6. We drop the time index, t, whenever doing so will not result in confusion.
7. For a lucid discussion on these issues, see also Romer (1991).
8. Corsetti and Roubini (1996) determine the optimal level of government spending within a model where, among others, the government is allowed to borrow and lend.
9. This is basically the well-known Mangasarian condition. Note, however, that this condition will not be satisfied in a planning problem in which (1) is maximized with respect to c, u, and g, subject to (2)–(5). In fact, one could not even rely on Arrow's weaker sufficiency condition, according to which the maximized Hamiltonian (with respect to c, u, and g) must be concave in k, for two reasons. First, the Hamiltonian may fail to achieve a maximum owing to the term $g^{1-\alpha}u^{1-\alpha}$ in (4) (recall that the product of two concave functions is not necessarily concave). Second, even if the maximized Hamiltonian is well defined, it may not be concave in k, since the production function (after substituting the maximizing value of g) may exhibit increasing returns to scale in k.
10. Note that, since g is taken as given by each individual firm, when we calculate $w = \partial y/\partial u$ and $r = \partial y/\partial k$ we differentiate with respect to u and k, respectively, holding g fixed, even though in equilibrium g depends on k and u (see the government budget constraint, (5)).
11. A similar condition is derived in Benhabib and Farmer (1994) in a model with capital and labor externalities. They note that the estimates of increasing returns provided by Caballero and Lyons (1992) imply that indeterminacy can occur with the values of ε well within an acceptable range.
12. It should be noted that the econometric methodology required to test this condition is quite different from that in typical empirical studies of dynamic labor supply and labor demand, since the latter studies typically ignore the presence of external effects which are essential to our argument.
13. We thank an anonymous referee for bringing this issue to our attention.
14. For a description of existing methods to solve problems such as the one encountered here, see Corsetti and Roubini (1996). The method followed here is similar to that in Chamley (1986).