Illegal Immigration, Factor Substitution and Economic Growth*

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1 Introduction

Illegal immigration is one of the most controversial and divisive issues in the developed world. This is evident even from the number of organizations and citizen groups that campaign either for or against it on the web. Moreover, every political party in developed countries seems to have included this issue in its agenda and proposes ways for the solution of the "problem."

On the other hand, an increasing number of scientific articles analyzes the economic consequences of illegal immigration. For example, Hazari and Sgro (2003), Moy and Yip (2006) and Palivos (2009) analyze the effects of illegal immigration in a one-sector optimal growth model and show that an increase in the number of immigrants can potentially increase the welfare of the representative household. Palivos and Yip (2010) analyze the same issue within a neoclassical growth model with two groups of workers, skilled and unskilled, under the assumption that domestic and immigrant workers are perfect

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substitutes. They show that although illegal immigration is a boon for the host country as a whole, there can be distributional effects. Liu (2010) concentrates on the welfare effects of illegal immigration within a dynamic general equilibrium model with search frictions. His basic model, where there is one type of domestic labor and immigrants and natives are imperfect substitutes, is calibrated to the US economy. He finds a U-shaped relationship between the population share of illegal immigrants and consumption per domestic resident in the long run. In an extended version of the model, where there are two types of domestic labor, namely skilled and unskilled, and illegal immigrants belong to the unskilled group, he finds distributional effects similar to those in Palivos and Yip (2010).

One of the crucial factors in the analysis of the effects of immigration is the degree of substitutability between natives and immigrants.[1] Illegal immigrants have usually acquired low levels of education. Moreover, they tend to have low language skills (see Peri and Sparber 2009). Hence, they constitute an imperfect substitute for native workers of similar observable characteristics, e.g., education level, and experience. In fact, the existing empirical literature has questioned the assumption of perfect substitutability between natives and immigrant workers (see, for example, the discussion in Ottaviano and Peri 2011), which is often employed in theoretical investigations. For example, when the substitutability between natives and immigrants is constrained to be the same across education and experience groups, Ottaviano and Peri (2011) estimate the elasticity of substitution to be about 20. When the substitutability is allowed to vary across education and experience groups, they find the elasticity of substitution between natives and immigrants to be 12.5 among low educated workers and 6.6 among young workers.

In this article we construct a neoclassical growth model of illegal immigration, which allows explicitly for imperfect substitutability between native and immigrant workers. Based on such a model, we investigate analytically and/or numerically the effects of illegal immigration on the average capital stock in the host economy as well as on the wage, income, and assets holdings of native workers. Moreover, as time elapses, immigrant workers acquire language skills, adjust to the new culture and become assimilated into the new society. Thus, it is reasonable to assume that the elasticity of substitution between immigrant and domestic labor increases over time. Hence, we also analyze the effects of

[1]This point is particularly emphasized in Ottaviano and Peri (2011), who, based on their estimates (see below), find that in the long run immigration has a small positive effect on US average native wages of about 0.6%.

a change in the elasticity of substitution between immigrant workers and natives for any given immigration ratio. We do this by employing the normalization technique introduced by La Grandville (1989) and advanced by Klump and de La Grandville (2000), Klump and Preissler (2000) and Papageorgiou and Saam (2008). We also analyze the impact of a change in the elasticity of substitution between skilled and unskilled labor.

The remainder of the article is organized as follows. Section 2 analyzes the effects of changes in the immigration ratio and in the elasticity of substitution between natives and immigrants in a basic model, where there is only one type of domestic labor. Section 3 examines similar issues in a model with two types of domestic labor. Section 4 summarizes the results and draws conclusions.

2 A Model of Illegal Immigration with One Type of Domestic Labor

The primary purpose of our paper is to analyze the effects of illegal immigration on the distribution of income and wealth in the host economy, where natives and migrants are imperfect substitutes in production. Of course, the analysis of issues pertaining to the distribution of income presupposes the presence of heterogeneous agents. Nevertheless, to understand some of the mechanisms at work and to establish notation, it is useful to start with an economy that is inhabited by just one type of labor; a more elaborate version of this model is considered in the next section.

2.1 The Model

Consider a Solow-type economy in which the production process is described by the function

$$Y = \Theta K^{\alpha} N^{1-\alpha}, \tag{1}$$

where Y denotes total output, K denotes aggregate capital, and N is an aggregate input of domestic (L) and immigrant labor (M).[2] Specifically,

$$N = B\left[\mu M^{\beta} + (1-\mu)\, L^{\beta}\right]^{\frac{1}{\beta}}, \quad \beta \leq 1. \tag{2}$$

[2]Since the elasticity of substitution between capital and aggregate labor is one, the model analyzed here cannot result in unbounded growth (see Solow 1956 and Palivos and Karagiannis 2010).

The elasticity of substitution between domestic and immigrant labor, denoted by σ, is defined as

$$\sigma = \frac{\partial \left(M/L\right)}{\partial \left(MPM/MPL\right)} \frac{MPM/MPL}{M/L} = \frac{1}{1-\beta}, \tag{3}$$

where, throughout the paper, MPX denotes the marginal product of factor X. Obviously, as β approaches one, the elasticity of substitution σ approaches infinity and L and M become perfect substitutes in production.

Substituting (2) into (1) yields the following two-level production function:

$$F\left(K, M, L\right) = AK^{\alpha}\left[\mu M^{\beta} + \left(1-\mu\right)L^{\beta}\right]^{\frac{1-\alpha}{\beta}}, \tag{4}$$

which in intensive form can be written as

$$f\left(k, m\right) = F/L = Ak^{\alpha}\left[\mu m^{\beta} + \left(1-\mu\right)\right]^{\frac{1-\alpha}{\beta}}, \tag{5}$$

where $A \equiv \Theta B^{1-\alpha}$, $k = K/L$ is capital per domestic worker and $m = M/L$ is the immigration ratio (number of immigrants per domestic citizen). For simplicity, we abstract from population growth.

All immigration $M \geq 0$ is assumed to be illegal (undocumented). Moreover, the following two assumptions are intended to characterize illegal immigration. First, there is a cost associated with the employment of an undocumented immigrant. If caught employing an illegal immigrant, an employer must pay a fine to the government. We denote the expected value of such a fine by γ (= the probability of being caught employing an illegal immigrant times the fine). Second, illegal immigrants do not save in terms of assets located in the host country. Instead, they send all their savings abroad.

This is a private ownership economy, where all resources (besides foreign labor of course) and firms are owned by households. Obviously, the constant returns to scale (CRS) property of the production function allows us to assume that there is only one firm, whose shares are held by domestic consumers. We also assume that all markets are competitive.

The income (y) of each domestic citizen consists of her capital income (rk), where r is the real interest rate, her labor income (w_L) and government transfers (τ).[3] Thus,

$$y = rk + w_L + \tau. \tag{6}$$

[3] Note that in general $y \neq Y/L$; the former denotes the income of each domestic citizen, while the latter is total output produced per domestic citizen. In other words, Y includes immigrant's income. The two variables are equal to each other only when there is no immigration ($M = 0$).

Furthermore, each domestic citizen saves a constant fraction $s \in (0,1)$ of her income (y).[4] Thus, savings per domestic citizen are sy.

The representative firm, which is assumed to be risk neutral, maximizes its expected profit

$$\Pi = F(K, L, M) - rK - w_L L - w_m M - \gamma M - \delta K, \tag{7}$$

where w_m is the wage paid to illegal immigrants and δ is the depreciation rate. The only source of uncertainty is with regard to the probability of being caught employing an illegal immigrant (recall that γ denotes the expected value of the fine). The first-order necessary conditions with respect to the three inputs are

$$F_K = r + \delta, \qquad F_L = w_L, \qquad F_M = w_m + \gamma, \tag{8}$$

where a subscript attached to a function symbol denotes the partial derivative with respect to that argument, e.g., $F_K = \partial F / \partial K$. Differentiating (5), we have

$$\alpha \frac{f}{k} = r + \delta, \quad (1 - \alpha - \pi) \frac{f}{m} = w_m + \gamma, \quad \pi f = w_L, \tag{9}$$

where π denotes the *income* share of domestic labor $(= w_L \times L/Y)$.

The government raises revenue (R) by imposing fines on firms that employ undocumented immigrants. As mentioned above, the expected value of the fine, which equals the probability that the firm is caught employing an illegal immigrant times the value of the fine, is γ. Hence, the government raises total revenue equal to

$$R = \gamma M.$$

We assume that this revenue is distributed back to the households in a lump-sum manner so that, at the end of each period, the government budget is balanced.

As we are interested in studying the effects of immigration when the substitutability of labor varies, we need to normalize the CES production function (5)[5]. To this end, we define the baseline point by choosing the marginal rate of substitution $\bar{\omega}$ (or, equivalently,

[4] For an analysis of the conditions that make the saving rate constant and render the Solow model isomorphic to the Ramsey model, see, among others, Litina and Palivos (2010).

[5] The CES function, when written in the traditional way, is inconsistent in units. This inconsistency disappears if one carefully identifies the two constants of integration involved in the original (Arrow et al. 1961) derivation of the CES function, as implied from a particular concave power function between the wage rate and income per person (see La Grandville 2009). In that sense, normalization is necessary before proceeding to our analysis.

the ratio of MPM to MPL), the level of income per capita $\bar{f}$, the capital-labor ratio $\bar{k}$, and the immigration ratio $\bar{m}$ to solve for:[6]

$$A(\beta) = \frac{\bar{f}}{\bar{k}^{\alpha}} \left[\frac{1 + \bar{m}^{1-\beta}\bar{\omega}}{1 + \bar{m}\bar{\omega}} \right]^{\frac{1-\alpha}{\beta}},$$

and

$$\mu(\beta) = \frac{\bar{m}^{1-\beta}\bar{\omega}}{1 + \bar{m}^{1-\beta}\bar{\omega}}.$$

After normalization, we have

$$f(k, m; \sigma) = A(\sigma) k^{\alpha} \left[\mu(\sigma) m^{\beta} + (1 - \mu(\sigma)) \right]^{\frac{1-\alpha}{\beta}}. \tag{10}$$

Note that the labor share is independent of k:

$$\pi(\sigma, m) = \frac{MPL \times L}{F} = \frac{1-\alpha}{1 + \bar{m}^{1-\beta}\bar{\omega}m^{\beta}} = \frac{(1-\alpha)(1-\mu)}{\mu m^{\beta} + 1 - \mu}, \tag{11}$$

and hence

$$\bar{\pi} = \frac{1-\alpha}{1 + \bar{m}\bar{\omega}}.$$

Differentiating (11), we have

$$\frac{\partial \pi}{\partial m} = -\frac{(1-\alpha)(1-\mu)\beta\mu m^{\beta-1}}{(\mu m^{\beta} + 1 - \mu)^2} < 0,$$

$$\frac{\partial \pi}{\partial \sigma} = \frac{\pi}{\sigma^2} \frac{1-\alpha-\pi}{1-\alpha} \ln \frac{\bar{m}}{m} \overset{<}{\underset{>}{}} 0 \text{ iff } m \overset{>}{\underset{<}{}} \bar{m}. \tag{12}$$

An increase in immigration reduces the income share of domestic workers. If we start the analysis from an initially no-migrant situation so that the normalized migrant-population ratio is taken to be zero, i.e., $\bar{m} = 0$, then we always have $m > \bar{m}$.[7] Thus, we assume that $\partial\pi/\partial\sigma < 0$ in the remainder of the analysis, i.e., an improvement in the substitutability between the two labor inputs raises the income share of the migrants starting from a lower migrant ratio. Next, assuming $m > \bar{m}$ and manipulating the production technology (10) yields[8]

$$\frac{\partial f}{\partial \sigma} = -\frac{1-\alpha}{\beta^2}\frac{f}{\sigma^2}\left[\frac{\pi}{1-\alpha} \ln\left(\frac{\bar{\pi}}{\pi}\right) + \frac{1-\alpha-\pi}{1-\alpha} \ln\left(\frac{1-\alpha-\bar{\pi}}{1-\alpha-\pi}\right) \right] > 0. \tag{13}$$

[6]Following common practice in the literature, we use the "over-bar" notation to denote normalized variables.

[7]Strictly speaking, in our calibrated exercises below, technically we cannot set $\bar{m} = 0$ as this would cause $\partial\pi/\partial\sigma$ to explode.

[8]Henceforth, whenever it does not cause any confusion and in order to simplify the notation, we simply denote with f the normalized CES function $f(k, m; \sigma)$ without specifying its arguments.

This is a standard result in the literature. An improvement in the substitutability between the two labor inputs raises the aggregate labor input and leads to more output.

Finally, we examine the effect of immigration on wages. From (9), we get

$$\frac{\partial w_m}{\partial m} = f_{mm} = -\frac{f}{m^2}\frac{1-\alpha-\pi}{(1-\alpha)\,\pi}\left[\alpha\,(1-\alpha-\pi)+(1-\beta)\,\pi\right] < 0,$$

$$\frac{\partial w_L}{\partial m} = f\frac{\partial \pi}{\partial m} + \pi f_m = (1-\alpha-\beta)\,\frac{\pi f}{m}\frac{\mu m^\beta}{\mu m^\beta + 1 - \mu}.$$

By diminishing returns, the wage of illegal migrants falls when more migrants come into the country; but the effect on the wage of native workers is ambiguous. This is because, on the one hand, diminishing factor returns due to the substitutability between workers raises the domestic wage when migrants come in. On the other hand, the increase in illegal migrants also reduces the native wage income share. In the special case where migrants and natives are perfect substitutes, both wages fall with illegal immigration since $w_L = w_m + \gamma$.

Next, we study the effect of a change in the substitutability between native and migrant workers on factor returns. According to (13), an increase in σ raises the returns to all factors due to an improvement in production efficiency. We call this the *efficiency effect* of factor substitution. With the Cobb-Douglas specification of (1), we know that the capital income share in output is constant and equals to α. Although the total share of income to all labor inputs $(L + M)$ remains unchanged, (12) informs us that there is a distribution effect on individual labor returns when σ changes. In particular, an increase in σ raises (lowers) the share of labor income of illegal migrants (native workers). We call this the *distribution effect* of factor substitution. As a result, we can conclude that the returns to illegal migrants must go up when natives and migrants are more substitutable in production; the effect on the wage of domestic workers, however, is ambiguous:

$$\frac{\partial w_m}{\partial \sigma} = \frac{1-\alpha-\pi}{m}\frac{\partial f}{\partial \sigma} - \frac{f}{m}\frac{\partial \pi}{\partial \sigma} > 0,$$

$$\frac{\partial w_L}{\partial \sigma} = \pi\frac{\partial f}{\partial \sigma} + f\frac{\partial \pi}{\partial \sigma}.$$

2.2 Steady-State Equilibrium

A balanced government budget requires that revenue equals transfers, which implies that the transfer received by each household is

$$\tau = \frac{R}{L} = \gamma\frac{M}{L} = \gamma m. \tag{14}$$

Furthermore, in equilibrium savings must equal investment. Measuring both variables per domestic citizen, we have

$$\begin{aligned} \dot{k} &= s\left[f(k,m;\sigma) - \delta k - w_m m\right] - \delta k \\ &= s\left[(\alpha+\pi) f(k,m;\sigma) - \delta k + \gamma m\right] - \delta k. \end{aligned} \tag{15}$$

In steady state, we have $\dot{k} = 0$ so that

$$sy(k^*, m; \sigma) = \delta k^*, \tag{16}$$

where $y(k,m;\sigma) \equiv \left[(\alpha+\pi) f(k,m;\sigma) - \delta k + \gamma m\right]$. It is straightforward to show that $y(k,m;\sigma)$ is strictly concave in k, $y(0,m;\sigma) \geq 0$, $\lim_{k\to 0} \partial y/\partial k > \delta/s$ and $\lim_{k\to\infty} \partial y/\partial k < \delta/s$. To ensure existence, uniqueness and stability of a positive steady-state level of capital stock, we further assume that $y(k,m;\sigma)$ is strictly increasing in k so that $(\alpha+\pi) f_k - \delta > 0$.

2.3 Changes in Wealth, Income and Consumption

Totally differentiating (16) yields

$$\frac{dk^*}{dm} = \frac{\frac{\partial y^*}{\partial m}}{\frac{\delta}{s} - \frac{\partial y^*}{\partial k^*}} > 0, \tag{17}$$

since

$$\frac{\partial y}{\partial m} = \gamma - m\frac{\partial w_m}{\partial m} = \left[(\alpha+\pi)(1-\alpha) - \beta\pi\right] \frac{1-\alpha-\pi}{1-\alpha}\frac{f}{m} > 0 \tag{18}$$

and $(\delta/s) > \partial y^*/\partial k^* = (\alpha+\pi)f_k - \delta$ (from the conditions for existence, uniqueness and stability). Since all steady-state variables remain in fixed proportion, that is, $y^* = (\delta/s)k^*$, $c^* = [(1-s)\delta/s]k^*$, the effects of immigration on income and consumption, y^* and c^*, are proportional to that on capital:

$$\frac{dy^*}{dm} = \frac{\delta}{s}\frac{dk^*}{dm} > 0, \qquad \frac{dc^*}{dm} = \frac{(1-s)\delta}{s}\frac{dk^*}{dm} > 0.$$

Thus, we conclude:

Proposition 1 *An increase in illegal immigration raises the steady-state per capita levels of capital, income and consumption.*

This occurs for two reasons. First, an increase in immigration generates more revenue through the exploitation of new immigrants (this is captured by the term γ in $\partial y^*/\partial m$; see equation 17). Second, the increase in immigration lowers the domestic wage and, thus, the wage paid to the existing immigrants (this is captured by the term $-m\left(\partial w_m/\partial m\right) = -mf_{mm} > 0$ in $\partial y^*/\partial m$). If $\gamma = 0$, then there is no exploitation and the first effect disappears. In addition, if initially there is no immigration (that is, if we are examining an incremental change in immigration starting from $m = 0$), then the second effect disappears as well. These two special cases not withstanding, we can conclude that, in a framework where there is only one type of labor, illegal immigration is in the long-run beneficial to domestic citizens, because it increases their steady-state level of consumption.

Regarding an increase in the substitutability between natives and migrants in production (σ), the effects on the steady-state levels of per capita capital and income are ambiguous:

$$\frac{dk^*}{d\sigma} = \frac{\partial y/\partial\sigma}{\frac{\delta}{s} - \frac{\partial y}{\partial k}} \Longrightarrow sign\left(\frac{dk^*}{d\sigma}\right) = sign\left(\frac{\partial y}{\partial\sigma}\right).$$

The ambiguity is due to the presence of two opposing effects at work, the distribution versus the efficiency effect:

$$\frac{\partial y}{\partial\sigma} = (\pi + \alpha)\frac{\partial f}{\partial\sigma} + f\frac{\partial\pi}{\partial\sigma}. \tag{19}$$

On the one hand, according to the efficiency effect, the increased substitutability of workers expands the aggregate labor input so that more income is generated. On the other hand, the distribution effect reduces (raises) the share that goes to native workers (immigrants) and hence reduces y. If the efficiency effect dominates, then we have $dk^*/d\sigma > 0$ so that an increase in the substitutability between the two types of workers raises the steady-state levels of per capita capital and income. To understand the intuition behind the ambiguity, we note that the efficiency effect raises income and hence savings that can be channeled into capital accumulation. Nevertheless, the fact that migrants do not save (or more precisely their savings are invested abroad) generates a leakage in income. If the distribution effect dominates so that the income share of migrants increases enough, then it is possible to have a decline in aggregate savings and hence in capital accumulation. Of course, if migrants save, then we have $y(k, m; \sigma) \equiv f(k, m; \sigma) - \delta k$ so that

$$\frac{\partial y}{\partial\sigma} = \frac{\partial f}{\partial\sigma} > 0 \Rightarrow \frac{dk^*}{d\sigma} > 0.$$

We summarize our findings regarding the effects of factor substitution in the following proposition:

Proposition 2 *If the efficiency effect dominates (is dominated by) the distribution effect of factor substitution, then an increase in the substitutability between immigrants and domestic workers raises (lowers) the steady-state levels of per capita capital, income and consumption.*

We close this subsection by providing a numerical example regarding the steady-state effects. Based on Ottaviano and Peri (2011), we specify σ to vary from 0.5 to 30. Moreover, based on calculations found in Palivos and Yip (2010), we set $\gamma = 0.25$. The value of $s = 0.15$ is based on Elwell (2010). The other parameter values are all common in the literature.

Baseline values for normalization	
$\overline{y} = 10$	$\overline{k} = 10$
$\overline{\pi} = 0.68$	$\overline{m} = 0.04$
Parameters for the benchmark model	
$\gamma = 0.25$	$\delta = 0.04$
$s = 0.15$	$\alpha = 0.3$
$m = 0.05$, 0.1 and 0.15	$\sigma \in (0.5, 30)$

We report the steady-state values of the wage rates, capital, and income in Figure 1. In the upper panels, we see that, for any given σ, an increase in m always raises k^* and y^*. But given m, an increase in σ lowers both k^* and y^*. This implies that the distribution effect of factor substitution dominates the efficiency effect. The effects on the wage rates are shown in the lower panels of Figure 1. We see that the effects on w_m confirm our comparative static findings: w_m is increasing in σ, but decreasing in m. Also, the effect of σ on w_L is negative so that again the distribution effect dominates the efficiency effect of factor substitution. Finally, the effect of m on w_L depends on the size of the elasticity of substitution, as predicted by our comparative statics. If σ is large enough so that the two labor inputs are good gross substitutes, then w_L is decreasing in m. Interestingly, however, if natives and immigrants are not very substitutable in production (i.e., σ is small enough), then w_L can increase with m.

2.4 Transitional Dynamics

Next, we analyze the response of capital to an increase in m during the transition to the new steady-state equilibrium. The dynamic behavior of k is described by equation (15).

Given our assumptions on the function $y(k, m; \sigma)$, we have

$$\frac{\partial \dot{k}}{\partial k} = s\frac{\partial y}{\partial k} - \delta = s\left[(\alpha+\pi)f_k - \delta\right] - \delta,$$

$$\frac{\partial^2 \dot{k}}{\partial k^2} = s\frac{\partial^2 y}{\partial k^2} = s(\alpha+\pi)f_{kk} < 0.$$

The curve $\dot{k}(k, m; \sigma)$ is shown in Figure 2. According to (18), an increase in m shifts up the $\dot{k}$ curve so that we have $dk^*/dm > 0$. Moreover,

$$\ddot{k} = \frac{d\dot{k}}{dt} = \frac{\partial \dot{k}}{\partial k}\frac{dk}{dt} = \left(s\frac{\partial y}{\partial k} - \delta\right)\dot{k} < 0,$$

since $s(\partial y/\partial k) - \delta < 0$ in the neighborhood of k^* and $\dot{k} > 0$ for $k < k^*$. Thus, we can deduce the dynamic adjustment of k (see Figure 3). The paths of consumption and income are similar. We conclude that in a society in which domestic and foreign labor are imperfect substitutes, illegal immigration is unambiguously beneficial to domestic citizens because it raises their consumption level both in the steady state and throughout the entire transition.

The effects of a change in σ are ambiguous due to the presence of the opposing efficiency and distribution effects. According to (19) and in terms of Figure 2, the $\dot{k}$ locus can shift either up or down when σ changes so that k^* can go in either direction. Based on our numerical example above, it is likely that the distribution effect dominates the efficiency effect. As a result, for a given level of illegal immigration, an increase in the substitutability between natives and migrants is likely to reduce capital accumulation, income and consumption in transition.

Figure 4 provides a numerical example for the effects during the transition of a change in m and σ on capital accumulation, using the same parameter values specified above. The upward-sloping path confirms the monotonic convergence of k over time. From the upper panel, illegal immigration raises the capital stock (and hence income and consumption) throughout the entire transition. On the contrary, an increase in the substitutability of workers lowers the capital stock (and hence income and consumption) over time in the transition towards the new steady state.

3 The General Framework

In the general framework the *aggregate* labor input H consists of two types of workers, unskilled labor N and (domestic) skilled labor S. Immigrants, who are all illegal, are unskilled. The input H is another CES aggregate:

$$H = \Phi \left[\theta N^{\eta} + (1-\theta) S^{\eta}\right]^{\frac{1}{\eta}}, \quad \eta \leq 1. \tag{20}$$

where N is given by (2). Final output Y is then produced with two aggregate inputs, capital K and labor H, using a Cobb-Douglas technology:

$$Y = \Theta K^{\alpha} H^{1-\alpha}. \tag{21}$$

Substituting (2) and (20) into (21) yields the following aggregate production function:

$$F(K, M, L, S) = AK^{\alpha} \left\{\theta B^{\eta} \left[\mu M^{\beta} + (1-\mu) L^{\beta}\right]^{\frac{\eta}{\beta}} + (1-\theta) S^{\eta}\right\}^{\frac{1-\alpha}{\eta}},$$

where $A \equiv \Theta\Phi^{1-\alpha}$. Moreover, the aggregate production function can be written in intensive form as

$$y = f(k, m, l) = Ak^{\alpha} \left\{\theta B^{\eta} \left[\mu m^{\beta} + (1-\mu) l^{\beta}\right]^{\frac{\eta}{\beta}} + (1-\theta)(1-l)^{\eta}\right\}^{\frac{1-\alpha}{\eta}},$$

where the lowercase variables k, m, and l are equal to the corresponding aggregate variable K, M, and L divided by the number of domestic citizens $L+S$. In particular, k and m denote the average capital stock and the immigration ratio, respectively.

Following the normalization concepts of Klump and de La Grandville (2000), as these were further elaborated by Papageorgiou and Saam (2008) for a two-level nested CES, we choose the baseline values in per capita term of $\overline{y}$, $\overline{k}$, $\overline{n}$, $\overline{h}$, $\overline{l}$, $\overline{m}$, $\overline{\pi}_M$ and $\overline{\pi}_N$, and calculate the normalized parameters of the CES production function (see equations (A.3)-(A.6) in the Appendix).

3.1 The Comparative Static Results of Factor Substitution

In this subsection we analyze the impact of a change in η and β on the nested inputs, aggregate output, factor shares and wages. First, define the nested inputs, namely, $n = N/(S+L)$ and $h = H/(S+L)$, and the *factor* share, $\widehat{\pi}_i$ of factor i, as the ratio of the factor's value contribution (or its marginal value product) to its related nested quantity. For example, the factor share of M in N can be written as

$$\widehat{\pi}_M = \frac{(\partial N/\partial M) \times M}{N} = \frac{\overline{\pi}_M (m/\overline{m})^{\beta}}{\overline{\pi}_M (m/\overline{m})^{\beta} + 1 - \overline{\pi}_M}.$$

Likewise, for the shares of L, N and S, we have

$$\widehat{\pi}_L = \frac{\partial N}{\partial L}\frac{L}{N} = 1 - \widehat{\pi}_M,$$

$$\widehat{\pi}_N = \frac{\partial H}{\partial N}\frac{N}{H} = \frac{\overline{\pi}_N (n/\overline{n})^\eta}{\overline{\pi}_N (n/\overline{n})^\eta + 1 - \overline{\pi}_N},$$

$$\widehat{\pi}_S = \frac{\partial H}{\partial N}\frac{S}{H} = 1 - \widehat{\pi}_N,$$

since both N and H exhibit constant returns to scale.

Next, we analyze the impact on the nested inputs n and h:

$$\frac{\partial n}{\partial \beta} = -\frac{n(1-\beta)^2}{\beta^2}\left[\widehat{\pi}_M \ln \frac{\overline{\pi}_M}{\widehat{\pi}_M} + (1-\widehat{\pi}_M)\ln \frac{1-\overline{\pi}_M}{1-\widehat{\pi}_M}\right] > 0,$$

$$\frac{\partial h}{\partial \eta} = -\frac{h(1-\eta)^2}{\eta^2}\left[\widehat{\pi}_N \ln \frac{\overline{\pi}_N}{\widehat{\pi}_N} + (1-\widehat{\pi}_N)\ln \frac{1-\overline{\pi}_N}{1-\widehat{\pi}_N}\right] > 0.$$

These results capture the standard efficiency effect of factor substitution, namely, better factor substitution is equivalent to an increase in factor inputs. This then yields a positive impact on the aggregate output per domestic citizen, $Y/(S+L)$:

$$\frac{\partial y}{\partial \eta} = \frac{\partial y}{\partial h}\frac{\partial h}{\partial \eta} > 0,$$

$$\frac{\partial y}{\partial \beta} = \frac{\partial y}{\partial h}\frac{\partial h}{\partial n}\frac{\partial n}{\partial \beta} > 0,$$

since $\partial y/\partial h > 0$ and $\partial h/\partial n > 0$.

Consider next the impact of η and β on factor shares. Simple differentiation yields

$$\frac{\partial \widehat{\pi}_M}{\partial \beta} = -\frac{\partial \widehat{\pi}_L}{\partial \beta} = (1-\widehat{\pi}_M)\,\widehat{\pi}_M \ln \frac{m}{\overline{m}} > 0 \text{ iff } m > \overline{m},$$

$$\frac{\partial \widehat{\pi}_N}{\partial \eta} = -\frac{\partial \widehat{\pi}_S}{\partial \eta} = (1-\widehat{\pi}_N)\,\widehat{\pi}_N \ln\left[\frac{n/\overline{n}}{(1-l)\,/\,(1-\overline{l})}\right] \gtrless 0 \text{ iff } \frac{n}{1-l} \gtrless \frac{\overline{n}}{1-\overline{l}}.$$

The effect of factor substitution on the factor shares depends on the relative abundance of the relevant factor. For instance, if migrants are relatively abundant (i.e., $m > \overline{m}$), then an improvement in factor substitution between native and immigrant unskilled labor leads to an increase in the employment of M. As a result, its factor share increases so that $\partial \widehat{\pi}_M/\partial \beta > 0$. This is the distribution effect emphasized in Xue and Yip (2011). Next, there is no effect of η on $\widehat{\pi}_M$ as the nesting structure does not allow for reversible impact of the upper-level substitutability on the lower-level factor shares, that is

$$\frac{\partial \widehat{\pi}_M}{\partial \eta} = 0.$$

Finally, the effect of β on $\widehat{\pi}_N$ is given by

$$\frac{\partial \widehat{\pi}_N}{\partial \beta} = \frac{\partial \widehat{\pi}_N}{\partial n}\frac{\partial n}{\partial \beta} = \frac{\eta}{n}\widehat{\pi}_N \left(1 - \widehat{\pi}_N\right)\frac{\partial n}{\partial \beta} \gtrless 0 \text{ iff } \eta \gtrless 0.$$

The effect of factor substitution on the upper-level factor share depends on the substitutability of the factors in the upper nesting level. For instance, the effect of β on $\widehat{\pi}_N$ depends on whether N and S are gross substitutes or gross complements in the formation of H (i.e., $\eta \gtrless 0$). If $\eta > 0$, then skilled and unskilled labor are gross substitutes in production. The efficiency effect of factor substitution expands the unskilled labor supply so that their share in aggregate labor increases.

Consider next the impact of η and β on the wage rates. We denote the marginal return of factor i by F_i. Moreover, profit maximization of the firm yields

$$F_K = r + \delta, F_M = w_m + \gamma, F_L = w_L, F_S = w_S,$$

where w_S is the real wage rate of skilled labor. Hence

$$w_S = F_S = \frac{\partial Y}{\partial H}\frac{\partial H}{\partial S} = (1-\alpha)\,\widehat{\pi}_S\frac{y}{1-l},$$

$$w_L = F_L = \frac{\partial Y}{\partial H}\frac{\partial H}{\partial N}\frac{\partial N}{\partial L} = (1-\alpha)\,\widehat{\pi}_N\widehat{\pi}_L\frac{y}{l},$$

$$w_m = F_M - \gamma = \frac{\partial Y}{\partial H}\frac{\partial H}{\partial N}\frac{\partial N}{\partial M} - \gamma = (1-\alpha)\,\widehat{\pi}_N\widehat{\pi}_M\frac{y}{m} - \gamma$$

$$r = F_K - \delta = \frac{\partial Y}{\partial K} - \delta = \alpha\frac{y}{k} - \delta.$$

If we define the *income* share of skilled labor in total output as π_S, we have

$$\pi_S = \frac{F_S S}{Y} = (1-\alpha)\,\widehat{\pi}_S.$$

Of course, for our results to have some meaning, the following restrictions on the parameter values must hold.

Assumption 1: The wage of a skilled worker should be higher than the wage of an unskilled, i.e.,

$$w_L < w_S \Rightarrow (1+\widehat{\pi}_L)\,\widehat{\pi}_N < 1.$$

Assumption 2: The wage rate of an immigrant must be positive:

$$w_m > 0, \quad \text{or} \quad \gamma < (1-\alpha)\,\widehat{\pi}_N \widehat{\pi}_M y/m.$$

Next, we define the wage premium $\widetilde{w} \equiv w_S/w_L$ and study the impact of factor substitution on it. We note that

$$\widetilde{w} \equiv \frac{w_S}{w_L} = \frac{1-\widehat{\pi}_N}{\widehat{\pi}_N \widehat{\pi}_L} \frac{l}{1-l}.$$

Hence, the effects on the wage premium must work through the factor shares $\widehat{\pi}_N$ and $\widehat{\pi}_L (= 1-\widehat{\pi}_M)$. The wage premium increases with the relative factor share of migrants $(\widehat{\pi}_M)$ but decreases with the relative factor share of unskilled labor $(\widehat{\pi}_N)$. This is because a higher $\widehat{\pi}_M$ leads to a lower wage for domestic workers, while a higher $\widehat{\pi}_N$ yields both a lower w_S and a higher w_L. Formally, the effects of factor substitution on the wage premium are

$$\frac{\partial \widetilde{w}}{\partial \beta} = \frac{l}{1-l} \frac{1-\widehat{\pi}_N}{\widehat{\pi}_N \widehat{\pi}_L} \left(\widehat{\pi}_M \ln \frac{m}{\overline{m}} - \frac{\eta}{N} \frac{\partial N}{\partial \beta} \right),$$

$$\frac{\partial \widetilde{w}}{\partial \eta} = -\frac{l}{1-l} \frac{(1-\widehat{\pi}_N)\,\widehat{\pi}_L \widehat{\pi}_N}{(\widehat{\pi}_N \widehat{\pi}_L)^2} \ln \left[\frac{n/\overline{n}}{(1-l)\,/\,(1-\overline{l})} \right] \gtrless 0 \text{ iff } \frac{n}{1-l} \lessgtr \frac{\overline{n}}{1-\overline{l}}.$$

If $n/(1-l) > \bar{n}/(1-\bar{l})$, the wage premium decreases as η increases. Better factor substitution between skilled and unskilled labor results in a higher factor share of unskilled workers if they are relatively abundant. This lowers their relative wage and so the wage premium rises. If migrants are relative abundant $(m > \overline{m})$, then an increase in β raises the factor share of migrants $(\widehat{\pi}_M)$ so that real wage for domestic workers (w_L) falls. If unskilled labor is complementary to skilled labor in production $(\eta < 0)$, this also leads to an increase of the relative factor of skilled workers $(\widehat{\pi}_S)$ and hence their real wage (w_S). Thus, the wage premium increases as β rises when $m > \overline{m}$ and $\eta < 0$. Thus, we have

Proposition 3 *If unskilled labor is relatively abundant, then better substitution between skilled and unskilled labor (higher η) results in a higher wage premium. If migrants are* *relative abundant and unskilled labor is complementary to skilled labor in production, then the wage premium increases as the substitutability between migrants and domestic unskilled workers goes up (higher β).*

3.2 Steady-State Analysis

The total income of each domestic group i is

$$Y_i = rK_i + w_i L_i + T_i,\ i = L, S,$$

where K_i is the total capital stock owned by group i and T_i denotes the total government transfers to the group. The asset accumulation function for each domestic citizen of type i is given by the net savings

$$\dot{k}_i = sy_i - \delta k_i,$$

where $y_i = rk_i + w_i + \gamma m$ and $k_i = K_i/i$, $y_i = Y_i/i$ are the per capita capital and output owned by members of group i, respectively. Using the CRS property of F, we can write

$$\begin{aligned} Y/\left(S+L\right) &= F\left(k, l, 1-l, m\right) \\ &= F_k k + F_L l + F_M m + F_H \left(1-l\right), \end{aligned}$$

where $k = K/\left(S+L\right) = lk_L + \left(1-l\right)k_S$.

The equation that describes the accumulation of the (weighted) average capital stock, k, is given by

$$\dot{k} = sy\left(k, m\right) - \delta k,$$

where

$$y\left(k, m\right) = f\left(k, l, m\right) - mw_M - \delta k.$$

In steady state $\dot{k} = \dot{k}_i = 0$ and hence we have

$$sy\left(k^*, m\right) = \delta k^*, \tag{22}$$

or,

$$\left[1 - \left(1-\alpha\right)\widehat{\pi}_N \widehat{\pi}_M\right] f\left(k^*, l, m\right) + \gamma m - \frac{\left(1+s\right)\delta}{s} k^* = 0.$$

Consider next the comparative statics results of a change on the immigration ratio m on the average level of the steady-state capital stock. Differentiating (22) yields

$$\frac{dk^*}{dm} = \frac{\frac{\partial y^*}{\partial m}}{\frac{\delta}{s} - \frac{\partial y^*}{\partial k^*}} > 0,$$

because

$$\frac{\partial y^*}{\partial m} = F_M - w_M - mF_{MM} = \gamma - mF_{MM} > 0,$$

$$\frac{\partial y^*}{\partial k^*} = F_K - mF_{MK} - \delta < \frac{\delta}{s}.$$

The effect of m on the steady state k^* remains qualitatively the same as in the case of perfect substitutes (see Palivos and Yip 2010).[9] Moreover, an increase in m will also raise the average steady-state levels of income and consumption.

On the other hand, the impact of a change in β or η on the steady state k^* is ambiguous since

$$\frac{dk^*}{d\beta} = \frac{\frac{\partial y^*}{\partial \beta}}{\frac{\delta}{s} - \frac{\partial y^*}{\partial k^*}},$$

where

$$\frac{\partial y^*}{\partial \beta} = \left[1 - (1-\alpha)\widehat{\pi}_N \widehat{\pi}_M\right] \frac{\partial f}{\partial \beta} - f(1-\alpha)\left(\widehat{\pi}_N \frac{\partial \widehat{\pi}_M}{\partial \beta} + \widehat{\pi}_M \frac{\partial \widehat{\pi}_N}{\partial \beta}\right). \tag{23}$$

Likewise, for the impact of η, we have

$$\frac{dk^*}{d\eta} = \frac{\frac{\partial y^*}{\partial \eta}}{\frac{\delta}{s} - \frac{\partial y^*}{\partial k^*}},$$

where

$$\frac{\partial y^*}{\partial \eta} = \left[1 - (1-\alpha)\widehat{\pi}_N \widehat{\pi}_M\right] \frac{\partial f}{\partial \eta} - f(1-\alpha)\widehat{\pi}_M \frac{\partial \widehat{\pi}_N}{\partial \eta}. \tag{24}$$

Factor substitution affects the steady-state average variables via two channels: efficiency and distribution. The efficiency effect is always positive since better factor substitution is equivalent to an increase in factor inputs. The efficiency effect is given by the first term on the right-hand side of (23) and (24). The distribution effect, however, is conditional on the relative abundance as well as the nature of substitutability between the two factors, i.e., whether they are gross substitutes or gross complements in production. In general the distribution effect is ambiguous and this makes ambiguous the overall steady-state effects.

Proposition 4 *In the steady state, the effects of illegal immigration on the average levels of per capita capital and income are positive. The effects of factor substitution on the average levels of per capita capital and income are ambiguous due to the conflicting effects of efficiency and distribution.*

[9] The effect of m on the transition of k is also qualitatively unchanged.

4 The Distribution of Wealth

In this section, we study the effects of illegal immigration on wealth distribution. In steady state, the holdings of assets by each member of group i are

$$k_i^* = \frac{w_i + \gamma m}{\delta/s - r}.$$

After substituting away the factor returns, we have

$$\alpha y \frac{k_i^*}{k^*} + F_i + \gamma m - \left(1 + \frac{\delta}{s}\right) k_i^* = 0. \tag{25}$$

Define the relative asset position of unskilled labor and skilled labor as κ_{Lt} and κ_{St} respectively, i.e.,

$$\kappa_{Lt} = \frac{l k_{Lt}}{k_t} \text{ and } \kappa_{St} = \frac{(1-l)\, k_{St}}{k_t}.$$

For the transitional dynamics, we have

$$\dot{k} = sf\left[1 - (1-\alpha)\,\widehat{\pi}_N \widehat{\pi}_M\right] - (1+s)\,\delta k + s\gamma m,$$

$$\dot{k}_S = sf\left[\alpha \frac{k_S}{k} + (1-\alpha)\,\frac{\widehat{\pi}_S}{1-l}\right] - (1+s)\,\delta k_S + s\gamma m,$$

$$\dot{k}_L = sf\left[\alpha \frac{k_L}{k} + (1-\alpha)\,\frac{\widehat{\pi}_N \widehat{\pi}_L}{l}\right] - (1+s)\,\delta k_L + s\gamma m.$$

From these three dynamic equations, we can solve for k, k_S and k_L.

Since it is not possible to derive analytical results, we rely on reasonable parameters values and numerical methods. More specifically, following the literature mentioned in Section 2 and Ottaviano and Peri (2011) for the elasticities of substitution, we adopt the following parameter values:

Baseline values for normalization		
$\overline{y} = 10$ $\overline{k} = 10$ $\overline{\pi}_M = 0.08$	$\overline{n} = 0.5$ $\overline{l} = 0.5$ $\overline{\pi}_N = 0.3$	$\overline{h} = 0.5$ $\overline{m} = 0.04$
Parameters for the benchmark model		
$\gamma = 0.25$ $s = 0.15$ $\sigma_{NS} \in (0.5, 3)$ and $\sigma_{LM} = 20$	$\delta = 0.04$ $l = 0.5$ $\sigma_{LM} \in (0.5, 30)$ and $\sigma_{NS} = 2$	$\alpha = 0.3$ $m = 0.05$, 0.1 and 0.15

where σ_{NS} is the elasticity of substitution between unskilled and skilled labor and σ_{LM} is the elasticity of substitution between domestic unskilled and illegal immigrant labor. Moreover,

$$\sigma_{NS} = \frac{1}{1-\eta}, \qquad \sigma_{LM} = \frac{1}{1-\beta}.$$

The parameter values specified above have to satisfy the restrictions given in Assumptions 1 ($w_L < w_S$) and 2 ($w_m > 0$). We report the findings in Figures 5 - 11. Figures 5 - 8 present the steady-state effects while Figures 9 - 11 illustrate the outcomes during the transition.

Figure 5 shows the steady-state effect of illegal immigration on the average level of per capita capital in the presence of factor substitution. According to the comparative statics, a rise in illegal immigration always raises the average level of per capita capital and this is indicated by the upward shift of the curves in Figure 5. On the other hand, the effect of factor substitution is in general ambiguous; nevertheless, our numerical exercise in the right panel of Figure 5 yield a negative impact in the case of an increase in the substitutability between skilled and unskilled labor. In other words, the distributive effect of σ_{NS} seems to dominate the efficiency effect in most of our baseline parameterization. Figures 6-7 summarize the steady-state effects of illegal immigration on type-i workers's wage rate and capital accumulation when the elasticities of factor substitution (and hence the parameters β and η) vary. Figure 6 shows that an increase in the elasticity of substitution between L and M (σ_{LM}) always reduces the unskilled wage but raises the wage of the skilled. This is because an increase in σ_{LM} works like an increase in aggregate unskilled labor N so that diminishing factor returns lead to a decrease (increase) in w_L (w_S). Figure 7 shows that an increase in the elasticity of substitution between N and S (σ_{NS}) instead reduces the skilled wage but raises the wage of the unskilled. This is because an increase in σ_{NS} allows for easier substitution between skilled and unskilled workers so that the returns between the two are converging. This in turn leads to a decrease in w_S and an increase in w_L. The intuition of the comparative static results on capital holdings, illustrated in Figures 6 and 7, follows the same reasoning. Finally, contrary to the findings of Palivos and Yip (2010), we have found that, given the elasticities of substitution, an increase in illegal migrants raises the wage rate and capital accumulation of both groups of domestic workers for most of the simulated cases in Figures 6-7 (shown by the shifts of the curves). In the presence of factor substitution, unskilled workers need not necessarily be worse off with illegal immigration. Only in the lower panel of Figure 7, we see that unskilled workers are worse off when the elasticity of substitution between skilled and unskilled labor (σ_{NS}) is relatively low. For instance, if $\sigma_{NS} < 1$, both the wage rate and capital holdings of the unskilled are decreasing with illegal immigration. Consequently, when skilled and unskilled workers are gross complements, illegal immigration worsens the income distribution between skilled and unskilled workers. This highlights the importance of factor substitution in understanding the effects of illegal immigration.

In general, as illegal immigration raises the average and individual capital holdings in the economy, the interesting question is to examine the overall distributive effect in the presence of factor substitution. According to Figure 8 (the right panels), given the substitutability between migrants and domestic unskilled labor, the relative asset holdings between skilled and unskilled workers diverge with illegal immigration. Nevertheless, the uneven distribution of asset holdings becomes more egalitarian as σ_{NS} rises. The intuition is as follows. An increase in σ_{NS} allows firms to substitute the relative abundant unskilled labor for the skilled; this then leads to a rise (fall) in the unskilled (skilled) workers' wage rate, per capita capital and relative capital share. Thus income distribution among the skilled and the unskilled is improved. From the left panels of Figure 8, we see that it is very likely that an increase in m and/or σ_{LM} worsens the domestic income distribution. The intuition is clear. As both an increase in σ_{LM} and m lead to an expansion of the unskilled labor supply N, their effects on income distribution are qualitatively equivalent. Only when σ_{LM} is low enough so that migrants and domestic unskilled workers are complementary in production, an increase in m may improve the domestic income distribution.

The transitional dynamic effects of a change in m, σ_{LM} and σ_{NS} on wealth distribution are shown in Figures 9, 10 and 11 respectively. According to Figures 9 and 10, an increase in m or σ_{LM} worsens the wealth distribution in transition due to diminishing factor returns. This is because these changes of the parameters lead to an expansion of the total unskilled labor supply N in the economy. Finally, as shown in Figure 11, an increase in σ_{NS} allows firms to better substitute the relative abundant unskilled labor for the skilled so that the uneven wealth distribution among the two groups is reduced. In summary, the analysis of the effects of illegal immigration, both in the steady state and in transition, can be very misleading if one ignores factor substitution.

5 Concluding Remarks

We have developed a Solow growth model with illegal immigration in which there exists either one or two types of domestic labor. We have also allowed for the possibility of imperfect substitution between unskilled and skilled labor as well as between natives and immigrants. Within such a framework we have analyzed the effects of an increase in immigration on the average capital stock, individual wages, asset holdings and the distribution of wealth. Moreover, utilizing the normalization technique, we have investigated in a systematic and comparable way the effects of factor substitution in the aforementioned labor inputs.

Our findings indicate that the effect of an increase in illegal immigration on the average levels of capital, consumption and income is positive. In other words, leaving distribution issues aside, an increase in illegal immigration makes a country as a whole better off. This result is in agreement with those found in previous studies. On the other hand, the effects of a change in the elasticities of substitution between different types of labor on the same variables are in general ambiguous, because of the presence of two often opposing effects: the efficiency and the distribution effects. Finally, contrary to previous results in the literature, we have shown that illegal immigration may not necessarily make the distribution of wealth more unequal and unskilled labor worse off. This is so because the end results depend on the elasticities of substitution between different types of labor. In fact, a lesson that has emerged throughout the article is that the distributional effects of illegal immigration depend crucially on the size of the aforementioned elasticities of factor substitution; assuming erroneously that immigrants and natives are perfect substitutes could lead to results that are not only over-estimated but also of the wrong sign.

Finally, let us conclude the paper with some interesting and relevant extensions of our analysis. One realistic extension is to allow for unemployment in the economy. In the model analyzed in this article, there is always full employment. Nevertheless, one of the arguments against illegal immigration is that it raises unemployment among native workers. Palivos (2009) studies the effects of illegal immigration in the presence of unemployment, when immigrants and natives are perfect substitutes. In this case, immigrants replace domestic unskilled workers on a one-to-one basis. There he finds that "immigration increases unemployment, leaves the capital stock unchanged, and decreases consumption and welfare" (p.140). Nevertheless, if we allow for factor substitution so that natives and immigrants are imperfect substitutes (or even gross complements), then an increase in illegal immigration no longer exhibits a one-to-one negative effect on domestic employment. Consequently, the effects of illegal immigration can be very different from those in Palivos (2009). Also, within the Solow growth framework, where there is a fixed saving rate, savings is a fraction of total aggregate income. As pointed out in Xue and Yip (2011), in models with an optimizing saving behavior, savings may come either from capital or from labor income. Consequently, the distribution effects of factor substitution are different within these models. This then would affect the overall outcome of illegal immigration.

A Appendix: Normalization Procedure of the Two-Level Nested CES Production Function

Following the normalization procedures of Klump and de La Grandville (2000) as well as Papageorgiou and Saam (2008), we define the baseline point of the CES functions by choosing the intensive (deflated by $L+S$) baseline values $\overline{y}$, $\overline{k}$, $\overline{n}$, $\overline{h}$, $\overline{l}$ and $\overline{m}$. Denoting $\widehat{\pi}_M \equiv (\partial N/\partial M)(M/N)$ as the share of M in N and $\widehat{\pi}_N \equiv (\partial H/\partial N)(N/H)$ as the share of N in H, the baseline values are given by

$$\overline{\pi}_M = \frac{\mu \overline{m}^{\beta}}{\mu \overline{m}^{\beta} + (1-\mu)\,\overline{l}^{\beta}}, \tag{A.1}$$

and

$$\overline{\pi}_N = \frac{\theta \overline{n}^{\eta}}{\theta \overline{n}^{\eta} + (1-\theta)\left(1-\overline{l}\right)^{\eta}}. \tag{A.2}$$

Next, we can use (A.1) and (A.2) to solve for the parameters μ and θ:

$$\mu(\beta) = \frac{\overline{\pi}_M \overline{l}^{\beta}}{\overline{\pi}_M \overline{l}^{\beta} + (1-\overline{\pi}_M)\,\overline{m}^{\beta}}, \tag{A.3}$$

$$\theta(\eta) = \frac{(1-\overline{\pi}_N)\left(1-\overline{l}\right)^{\eta}}{\overline{\pi}_N \overline{n}^{\eta} + (1-\overline{\pi}_N)\left(1-\overline{l}\right)^{\eta}}. \tag{A.4}$$

Also, from the aggregate unskilled labor function (2), we can solve for the parameter B:

$$B(\beta) = \left[\overline{\pi}_M \overline{m}^{-\beta} + (1-\overline{\pi}_M)\,\overline{l}^{-\beta}\right]^{\frac{1}{\beta}} \overline{n}. \tag{A.5}$$

Finally, combining these parameters with the normalized version of the CES production function, we can solve for A:

$$A(\eta) = \left[\overline{\pi}_N \overline{n}^{-\eta} + (1-\overline{\pi}_N)\left(1-\overline{l}\right)^{-\eta}\right]^{\frac{1-\alpha}{\eta}} \overline{h}^{1-\alpha}\overline{k}^{-\alpha}\overline{y}. \tag{A.6}$$

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