*Oxford Economic Papers* 57 (2005), 34–50 
doi:10.1093/oep/gpi007

# Optimal monetary policy with heterogeneous agents: a case for inflation

By Theodore Palivos

Department of Economics, University of Macedonia, 156 Egnatia Street, Thessalonica 540 06, Greece; e-mail: tpalivos@uom.gr

This paper analyses the role of monetary policy in an overlapping-generations monetary growth model with two types of agents, who exhibit a different degree of altruism towards their descendants. It is shown that changes in the money growth rate have significant distributional effects. Furthermore, the optimal rate of monetary expansion is, in general, higher than the one implied by the Friedman rule and may, in fact, yield a small but positive rate of inflation, even though capital is invariant to changes in the money growth rate. Finally, this optimal rate of monetary expansion takes higher values as the society's aversion towards inequality increases.

JEL classification: O42, E58

## 1. Introduction

What is the optimal rate of money growth and therefore of inflation? This is an important question that has been studied extensively in the field of monetary economics.[1] As it is the case with many other questions in the fields of macroeconomics and monetary theory, the answer depends on whether agents are altruistic towards their descendants; that is, it depends on whether agents exhibit a bequest motive or are selfish.

In a classic paper, written 36 years ago, Friedman (1969) has argued that an optimal monetary policy calls for deflation at a rate equal to the real rate of interest (Friedman rule). This argument has been confirmed in a variety of monetary growth models, in which consumers have an operative bequest motive and, hence, in effect infinite horizon. The reason is that in these types of models, an increase in the rate of monetary expansion decreases the value of real money balances and either leaves the steady-state capital stock unchanged, as Friedman implicitly assumed, or decreases it (reverse Tobin effect).[2] Hence, there is

[1] A comprehensive survey can be found in Woodford (1990).

[2] For a model in which the capital stock is invariant to changes in the money growth rate see Sidrauski (1967). Examples that exhibit a reverse Tobin effect include Brock (1974) and Abel (1985).

no trade-off between real money balances and capital and the optimal policy calls for the lowest possible money growth which is consistent with the existence of a long-run monetary equilibrium. This leads to the Friedman-rule rate.[3]

In contrast, in monetary growth models with selfish agents the optimal money growth rate is greater than the Friedman-rule rate and, under certain conditions, results in a fixed stock of fiat money. This occurs because, with finite lifetimes and zero bequests, money and capital are substitutes. Specifically, an increase in the money growth rate leads to an increase in the inflation rate (a decrease in the rate of return on money). This induces a decrease in real money balances and an increase in the steady-state capital stock (Tobin effect). The consideration of the optimal monetary policy must then take into account this trade-off (see, for example, Weiss, 1980; Drazen, 1981; Freeman, 1993).

This paper pursues a natural extension of the existing literature, namely the analysis of the optimum quantity of money in an environment where there are both altruistic and non-altruistic agents. For this purpose, it develops an overlapping-generations model with two types of agents who differ with respect to the degree of altruism towards their descendants. The high-altruism agents have an operative bequest motive and hence they are, in essence, infinitely lived. On the contrary, the low-altruism agents do not leave any bequests and, thus, behave as having a finite horizon and being non-altruistic.

This environment captures the redistributive effects of monetary policy. More specifically, since agents are differently situated, they hold different quantities of money. Thus, a steady monetary expansion, by generating inflation, will transfer wealth from agents who hold large money balances to agents that hold smaller balances. To the extend that such a redistribution transfers wealth from wealthy to poor agents, it will be desirable. The size of this effect, which is absent in the homogeneous agent literature and works against the effects upon the incentives to hold money, will depend on the relative size of each group and the society's aversion towards inequality. In general, the rate of money growth that maximizes a non-extreme welfare function is higher than the one implied by the Friedman rule and may, in fact, yield a positive rate of inflation.

## 1.1 Related literature

The study of the optimal quantity of money in the case of heterogeneous agents has already been suggested by William A. Brock (1974, 1990). In the former paper Brock concluded

> Let us close this paper with three suggestions for further work... Second, heterogeneous consumers should be introduced... It may turn out that one consumer is best off with [money] growth factor $\sigma_1$ whereas another is best off with $\sigma_2$ (1974, p.775).

[3] For an up-to-date discussion on the implementation of the Friedman rule see Ireland (2003).

Also, in the latter article Brock wrote

> Consider the case of $h$ households each with a degree of caring about their descendants $b_1 < b_2 < \ldots < b_H \ldots$ The point is that I have not seen a careful study of the optimum quantity of money in a heterogeneous agent... OG model...(1990, pp. 288–89).

There have been a few attempts in the literature to determine the optimal monetary policy in the presence of various types of heterogeneity, albeit different from the one suggested by Brock and analysed in this paper. Levine (1991) develops a model in which two types of infinitely lived agents shift randomly back and forth between being buyers and sellers (the two types value the consumption of a single good differently). Each agent's type is private information and transfers are independent of an agent's type. It is shown that an expansionary monetary policy (positive money growth rate) improves welfare because it provides insurance to buyers whose purchasing power is low as a result of bad shocks. In contrast, in the present paper there is no uncertainty, the heterogeneity is, in essence, with respect to the time horizon (infinite or finite), and both cases of private and no private information are considered.

Smith (1998), on the other hand, considers an overlapping generations model without any bequest motive in which each generation consists of two groups of individuals: 'small savers' and 'capitalists'. Furthermore, there is an indivisibility in capital investment, which precludes small savers from acquiring capital; thus, they save only in terms of money. The paper shows that if the initial equilibrium is dynamically inefficient then there may be many monetary policies (and inflation rates) that are consistent with a zero nominal interest rate. In this paper, there is no indivisibility and both types are *ex ante* altruistic towards their descendants; furthermore, the equilibrium is always dynamically efficient.

Finally, Shi (1999) develops a turnpike model with infinitely many goods to examine the redistributive role of monetary policy. The heterogeneity is introduced in the model by assuming that in each current and future market agents alternate between being buyers and sellers (the two types have different endowments of goods and money; in particular, monetary transfers are made only to buyers). Furthermore, there are two frictions in the model. The first is a within-market friction emanating from the fact that in each market $i$ both buyers and sellers have a desire for good $i$ but only sellers have it. This friction is (implicitly) present in most monetary models and is what renders money a valuable medium of exchange. If this were the only friction then the Friedman rule would have been optimal. Nevertheless, a second friction is introduced by assuming that sellers' marginal utility from consuming good $i-1$ is lower than buyers'. This friction calls for an expansionary monetary policy, since such a policy shifts capital from cash-constrained agents (buyers) to non-constrained agents (sellers). The optimal money growth rate is then the one that balances these two frictions and is greater than the Friedman rule. In this paper, a similar redistributive role of monetary policy is present.[4] However, the Friedman rule is, in general, not optimal, despite the fact that there is no cross-market friction.

The remainder of the paper is organized as follows. Section 2 presents the basic theoretical framework. Section 3 characterizes the optimal rate of monetary expansion in the case of equal transfers. Section 4 considers the case where the government makes unequal transfers to the two types of agents. Section 5 concludes the paper.

## 2. The model

Consider an overlapping-generations model with two types of agents who differ with respect to their degree of altruism towards their offspring, $\beta_j$, $j = L, H$, where $0 \le \beta_L < \beta_H < 1$. Let $q$ denote the fraction of agents with a degree of altruism $\beta_H$ and, consequently, $1 - q$ the fraction with a degree of altruism $\beta_L$. All agents live for two periods. The first period they work and earn the competitive wage while the second period they are retired. While young they hold their income either in terms of money or in terms of capital. For simplicity, I assume that agents do not consume in the first period.[5] More specifically, each agent derives utility from real money balances when young ($m$) and consumption when old ($c$). Thus, the lifetime utility of an agent of type $j$ who was born in period $t$ is given by

$$U_{jt} = L\left(m_{jt}\right) + V\left(c_{jt+1}\right) + \beta_j U_{jt+1}, \quad j = L,\ H. \tag{1}$$

To ensure an interior solution in the optimization problems considered below, I assume that $L$ is concave, twice continuously differentiable, with $\lim_{m \to 0} L'(\cdot) = \infty$, $L'(\cdot) > 0$ if $m < \bar{m}$ and $L'(\cdot) = 0$ if otherwise where $0 < \bar{m} < \infty$. On the other hand, $V$ is strictly concave, strictly increasing and twice continuously differentiable, with $\lim_{c \to 0} V'(\cdot) = \infty$ and $\lim_{c \to \infty} V'(\cdot) = 0$.

The population size is constant. I also denote the interest factor on capital by $R$ and that on money by $\gamma$; that is, $R_t \equiv 1 + r_t$, where $r_t$ is the net interest rate, and $\gamma_t \equiv p_{t-1}/p_t \equiv 1/(1 + \pi_t)$, where $p$ and $\pi$ stand for the price level and the inflation rate, respectively. As soon as individuals become old they exchange their money and capital for the consumption good. In addition, in the second period they receive a lump-sum money transfer, $a$, from the government.[6] Thus, each

[4] The potentially redistributive role of an expansionary monetary policy is also described in Woodford (1990).

[5] As in Freeman (1993), relaxing this assumption, by letting agents consume in both periods, does not alter the main implications of the paper. Kim (2001) uses a similar model to analyze the optimum quantity of money in the case of two-sided altruism.

[6] For a discussion on the assumption that the transfers are made to the old generation, see Drazen (1981) and Bhattacharya and Haslag (2001). There is also a large literature on the optimal inflation in the presence of distortionary taxation. For recent treatments see, among others, Woodford (1990), Chari *et al.* (1996), Correia and Teles (1996, 1999), Mulligan and Sala-i-Martin (1997), and Chari and Kehoe (1999).

agent faces the following budget constraints in the first and second period of life respectively:

$$m_{jt} + s_{jt} \leq w_t + b_{jt} \tag{2}$$

$$c_{jt+1} + b_{jt+1} \leq s_{jt}R_{t+1} + m_{jt}\gamma_{t+1} + a_{jt+1}, \qquad j = L, H \tag{3}$$

where $s$, $b$, and $w$ denote savings, bequests, and the wage rate.

Agents maximize their lifetime utility (eq. 1) with respect to $c_{jt+1}$, $m_{jt}$, $s_{jt}$, and $b_{jt+1}$, subject to the budget constraints (eqs 2 and 3) and the non-negativity constraint $b_{jt+1} \geq 0$, taking prices $w_t$, $R_{t+1}$, $\gamma_{t+1}$, *and* the transfer $a_{jt+1}$ as given. The first-order necessary conditions are

$$V'\left(c_{jt+1}\right) = \lambda_{j2t} \tag{4}$$

$$L'\left(m_{jt}\right) = \lambda_{j1t} - \lambda_{j2t}\gamma_{t+1} \tag{5}$$

$$\lambda_{j1t} = \lambda_{j2t}R_{t+1} \tag{6}$$

$$-\lambda_{j2t} + \beta_j\lambda_{j1t+1} \leq 0, \quad b_{jt+1} \geq 0, \quad \text{with complementary slackness} \tag{7}$$

the budget constraints, and a typical transversality condition. The variables $\lambda_{j1t}$ and $\lambda_{j2t}$ are the non-negative Lagrangean multipliers associated with the constraints (2) and (3), respectively. Notice that combining eqs (4), (5), and (6) yields $(R_{t+1} - \gamma_{t+1})V'(c_{jt+1}) = L'(m_{jt})$, or $R_{t+1} \geq \gamma_{t+1}$, a condition that I henceforth impose throughout the paper. Thus, capital cannot be dominated in rate of return by money.

The aggregate economy consists also of a large number of identical firms. Each firm operates according to a neoclassical production function $y = f(k)$, where $y$ and $k$ denote output and capital per worker. Furthermore, factor markets are competitive and capital depreciates fully within one period. Thus, $w_t = f(k_t) - k_tf'(k_t)$ and $R_t = f'(k_t)$.

Next consider the money market. Here, I assume that the government distributes the newly issued money to all agents equally (that is, $a_{Ht+1} = a_{Lt+1} = a_{t+1}$). Thus, the quantity of money issued in each period can be written, in real terms, as

$$qa_{Ht+1} + (1-q)a_{Lt+1} = a_{t+1} = (\mu - 1)\gamma_{t+1}\left[qm_{Ht} + (1-q)m_{Lt}\right] \tag{8}$$

where $\mu$ is the gross money growth rate. Moreover, equilibrium in the money market requires that

$$a_{t+1} + \gamma_{t+1}\left[qm_{Ht} + (1-q)m_{Lt}\right] = qm_{Ht+1} + (1-q)m_{Lt+1}.$$

The left-hand side represents the total supply of real money balances, which consists of newly issued balances distributed as transfer payments to the old (first term) and of the money carried over from the previous period by the currently old generation (second term). The right-hand side, on the other hand, is the demand for real money balances by the currently young generation. Combining the last two expressions yields

$$\mu\gamma_{t+1}\big[qm_{Ht} + (1-q)m_{Lt}\big] = qm_{Ht+1} + (1-q)m_{Lt+1}. \tag{9}$$

Finally, equilibrium in the goods market requires that the demand for goods in each period be equal to the supply.[7] Thus,

$$qc_{Ht+1} + (1-q)c_{Lt+1} + k_{t+2} = f(k_{t+1}). \tag{10}$$

## 3. The optimal money growth rate

Throughout the paper, I analyse the optimal monetary policy mainly in terms of $\gamma$ (the interest factor on money) The optimal inflation rate ($\pi$) follows then from $\gamma \equiv 1/(1+\pi)$ and the optimal money growth rate ($\mu$) from the steady-state version of (9), namely $\mu\gamma = 1$. Thus, consider the optimal value of $\gamma$, $\gamma^*$, defined as the value that maximizes a Bergson-Samuelson welfare function of the steady-state utility levels of both types of agents.[8] That is, $\gamma^*$ is the solution to the following problem:

$$\max_{\gamma} W = W(U_H, U_L) \tag{P}$$

subject to:

$$R = f'(k) \tag{11}$$

$$w = f(k) - kf'(k) \tag{12}$$

$$-1 + \beta_j R \leq 0, \quad b_j \geq 0, \quad \text{with complementary slackness} \tag{13}$$

$$(R-\gamma)V'\big(c_j\big) = L'\big(m_j\big) \tag{14}$$

$$s_j + m_j = b_j + w \tag{15}$$

$$c_j + b_j = Rs_j + \gamma m_j + (1-\gamma)\big[qm_H + (1-q)m_L\big] \tag{16}$$

$$qc_H + (1-q)c_L = [(R-1)k + w] \tag{17}$$

$$\gamma \leq R \tag{18}$$

[7] Using Walras's law, I ignore the equilibrium condition in the capital market, $qs_{Ht} + (1-q)s_{Lt} = k_{t+1}$.

[8] For a justification of this welfare criterion, see Woodford (1990) and Freeman (1993).

where $j = L, H$. Equations (11)–(17) follow from (2)–(10) and the equilibrium conditions in the factor markets, after imposing familiar steady state conditions. They characterize the steady-state equilibrium and determine $R$, $k$, $w$, $c_H$, $b_H$, $m_H$, $s_H$, $c_L$, $b_L$, $m_L$, and $s_L$ as stationary functions of $\gamma$. Thus, they act as constraints in the optimization problem. Before analysing the general case of heterogeneous agents, I briefly consider two special cases that have already been examined in the literature, one in which the economy consists of only altruistic and one in which it consists of only non-altruistic agents.

## 3.1 Altruistic agents

If the economy is populated only by altruistic agents then the steady-state equilibrium is characterized by (11)–(18) with $q = 1$. There are two cases to consider: (i) $1 = R\beta$ and $b \geq 0$ and (ii) $1 > R\beta$ and $b = 0$.[9] If $1 = R\beta$ and $b \geq 0$ then (11) results in the familiar modified golden rule, $1/\beta = f'(k)$. This implies a dichotomy between the capital accumulation decision and the holding of real money balances decision. Thus, changes in $\gamma$ have no effect on the steady-state capital stock ($\mathrm{d}k/\mathrm{d}\gamma = 0$). Furthermore, (17) implies that consumption is also independent of $\gamma$; money is superneutral.

Next consider the value of $\gamma^*$. The maximization problem (P) becomes:

$$\max_{\gamma} U = \frac{V(c) + L(m)}{1 - \beta}$$
$$\text{subject to} \quad (11) - (18)$$

Given money superneutrality, simple differentiation implies that the optimal value of $\gamma$ is the one that yields $L'(m) = 0$ or, using (8), $\gamma^* = R$ (the Friedman rule). Thus, the maximum level of welfare is attained at the point where the rate of return on money, $\gamma$, equals the rate of return on other assets, $R$. Since changes in $\gamma$ have an impact only on real money balances, it is optimal to satiate agents with money, which occurs when the opportunity cost of holding money is zero ($R - \gamma = 0$). Furthermore, $R = 1/\beta > 1$ implies that $\gamma^* > 1$ ($\pi^* < 0$).

If $1 > R\beta$ and $b = 0$ then agents are bequest-constrained, behaving in essence as being non-altruistic, a case that I turn to next.

## 3.2 Non-altruistic agents

If all agents are non-altruistic ($q = \beta_L = 0$) then the steady-state equilibrium is described by (11), (12), $b_L = 0$, and (14)–(18). In this case, $c$, $k$, and $m$ are determined simultaneously. Simple differentiation yields $\mathrm{d}k/\mathrm{d}\gamma = -V'/\Delta < 0$ if $R > \gamma$ ($= 0$ if $R = \gamma$), where $\Delta \equiv (R - 1)(\gamma - R)V'' - \big(kf'' + 1\big)L'' - f''V' > 0$, for the

[9] Whenever there exists only one type of agents, I omit the subscripts $L$ and $H$.

equilibrium to be stable. Thus, unlike the case of an altruistic economy, an increase in the rate of return on money induces substitution against capital accumulation (Tobin effect). Solving the problem

$$\max_{\gamma} U = V(c) + L(m)$$
$$\text{subject to} \quad (11),\ (12),\ b_L = 0,\ \text{and}\ (14)-(18)$$

yields

$$\gamma^* = R + (R-1)\left(\frac{\mathrm{d}w/\mathrm{d}\gamma}{\mathrm{d}m/\mathrm{d}\gamma} - 1\right). \tag{19}$$

For the purpose of this paper, the case where the first period's income is fixed (that is, $\mathrm{d}w/\mathrm{d}\gamma = 0$) is of particular importance.[10] This is so, because, as shown below, when heterogeneous agents co-exist the money growth rate has again no impact on the capital stock and hence $w$ is independent of $\gamma$. It follows from (19) that in this case $\gamma^* = 1$ (zero inflation rate). To see precisely the reason behind this result, notice, from (14), that agents set their marginal rate of substitution of real money balances for consumption equal to the opportunity cost of holding money, $R - \gamma$. In doing so, they take the monetary transfer from the government as given. Nevertheless, the actual transfer depends on their money holdings. Hence, $R - \gamma$ does not in general reflect the actual cost of holding money. Indeed, if one internalizes the transfer then one obtains a trade-off between $c$ and $m$ equal to $R - 1$; to see this notice that combining $q = 0$, $b_L = 0$, (15), and (16) yields the lifetime budget constraint: $c_L + (R-1)m_L = Rw$. Optimality then requires that there is no divergence between the actual cost of holding money and the one perceived by consumers, i.e., $R - 1 = R - \gamma$. This leads to $\gamma^*$ equal to 1.

### 3.3 Heterogeneous agents

We are now ready to analyse the main results of the paper, which pertain when $0 < q < 1$. Inequality (13), applied to each type, implies that, in steady-state, $\beta_H \leq 1/R$ and $\beta_L \leq 1/R$, where $\beta_L < \beta_H$. There exist then two possibilities. The first possibility is $\beta_L < \beta_H = 1/R$, in which case $b_L = 0$ and $b_H \geq 0$; the agents with the lowest degree of altruism are thus bequest-constrained and, in essence, behave as being non-altruistic. The second possibility is $\beta_L < \beta_H < 1/R$, with $b_L = b_H = 0$. This case, however, is not interesting, and I will henceforth ignore it, since both types behave as being non-altruistic. In other words, in the latter case there is no heterogeneity among agents.

[10] This is also the case analysed in Freeman (1993) and is similar to that of an endowment economy.

*Proposition 1* The capital stock is invariant to changes in the rate of return on money ($\mathrm{d}k/\mathrm{d}y = 0$).

*Proof* Combining eqs (11) and (13) (with equality) yields $1/\beta_H = f'(k)$ (the modified golden rule). From this, it follows immediately that $\mathrm{d}k/\mathrm{d}y = 0$. □

Hence, even in the case of heterogeneous agents, capital is invariant to changes in the money growth rate. In fact, notice that this result does not depend on the relative number of high-altruism agents ($q$). Even if there exist very few high-altruism agents, they will adjust their savings so that the modified golden rule holds.[11]

Next, I compare the steady-state consumption, real money balances, and utility levels of the two groups.

*Lemma 1* In steady-state: (i) the high-altruism agents have a higher level of consumption and hold an amount of real balances that is at least equal to that held by the low-altruism agents ($c_H > c_L$, $m_H \geq m_L$) and (ii) the high-altruism agents achieve a higher utility level than the low-altruism agents ($U_H > U_L$).

*Proof* (i) Suppose first that $R > \gamma$. Equation (14), applied to each type, yields $L'(m_H)/V'(c_H) = L'(m_L)/V'(c_L) = R - \gamma > 0$. Hence, if $c_H \gtrless c_L$ then $m_H \gtrless m_L$. It suffices therefore to show that $c_H > c_L$. By way of contradiction, suppose that $c_L \geq c_H$. It follows then from (16) that

$$Rs_L + \gamma m_L \geq Rs_H + \gamma m_H - b_H$$

or, by using (15),

$$(R - \gamma)m_L \leq (R - \gamma)m_H - (R - 1)b_H$$

which implies (since $R > \max\{1, \gamma\}$ and $b_H > 0$)$m_L < m_H$, and finally, as noted above, $c_L < c_H$. Next suppose $R = \gamma$. Then $m_L = m_H = \bar{m}$ and $c_H - c_L = (R - 1)b_H > 0$.[12] (ii) The proof of $U_H > U_L$ follows immediately from $c_H > c_L$, $m_H \geq m_L$, and $0 < \beta_L < \beta_H < 1$ since $U_j = [V(c_j) + L(m_j)]/(1 - \beta_j)$, $j = L, H$. □

Intuitively, the high-altruism agents are able to consume more and achieve a higher steady-state utility level for two reasons. First, the discount factor that applies to the utility of their descendants is higher; hence, *ceteris paribus*, the present value of their lifetime utility is higher. Second and more important, however, is the fact that

[11] Note, however, that money is non-superneutral since changes in the money growth rate and hence in $\gamma$ affect, among others, the consumption level of each type of agents, $c_H$ and $c_L$ In other words, changes in the growth rate of the nominal money supply alter the real equilibrium of the economy.

[12] Note that, for the sake of brevity, I have ignored the exceptional case where the non-negativity constraint is just binding, i.e., $R = 1/\beta_H$ and $b_H = 0$, because this is a knife-edge case.

they inherit from their parents and bequeath to their children $b_H$. This leaves for them a potentially additional income of $(R-1)b_H > 0$.

Let $\gamma_L^*$ and $\gamma_H^*$ denote the values of $\gamma^*$ preferred by the low- and high-altruism agents, respectively. These are the values of $\gamma^*$ that will be chosen by a social planner who cares about the utility of only one of the two groups (note, however, that, in contrast to the cases where $q=0$ or $q=1$, in this case the two groups co-exist).

*Proposition 2* (i) The low-altruism agents prefer a money growth rate which is higher than the Friedman-rule rate and yields a positive rate of inflation ($R > 1 > \gamma_L^* > 0$). (ii) The high-altruism agents prefer the Friedman-rule rate of money growth ($\gamma_H^* = R$).

*Proof* The proof of each part follows simply from the maximization of $U(c_j, m_j)$, $j=H, L$, subject to eqs (11)–(18). □

The interesting and somewhat surprising difference between these results and those in Subsection 3.2 is that when the two types co-exist the low-altruism agents prefer an interest factor on money, $\gamma$ which is less than one; that is, a positive inflation rate. Recall that non-altruists (who resemble the low-altruists in this section), when they live in isolation, prefer $\gamma=1$ (zero inflation rate). The reason actually for this result is quite simple. Inflation changes the real income of the low-altruism agents by $(\gamma-1)m_L$ (recall that they carry $m_L$ balances to the second period). At the same time, they receive a transfer from the government equal to $(1-\gamma)[qm_H + (1-q)m_L]$. The net impact is $q(1-\gamma)(m_H - m_L)$, which is positive if $\gamma < 1$. Put differently, since the money holdings of the low-altruism agents are lower than average (recall from Lemma 1 that if $\gamma < R$ then $m_L < m_H$ and hence $m_L < qm_H + (1-q)m_L$), a positive inflation rate raises their real wealth. On the other hand, the high altruism agents, who hold a higher than average amount of real money balances, experience a net change of $-(1-q)(1-\gamma)(m_H - m_L) < 0$ if $\gamma < 1$.[13]

In general, it is not possible to provide an analytical expression for the optimal value of $\gamma^*$. Nevertheless, the following proposition establishes some characterization of it.

*Proposition 3* (i) The value of the money growth rate that maximizes the social welfare function falls in between the rates preferred by the two groups. ($R = \gamma_H^* > \gamma^* > \gamma_L^*$) (ii) The higher the society's aversion towards inequality, the higher the optimal rate of monetary expansion is (or the closer $\gamma^*$ to $\gamma_L^*$ is).

*Proof* The proofs follow immediately from Lemma 1 and Proposition 2. □

[13] An example may help to illustrate this point further. Suppose that the two groups co-exist in equal proportions. Suppose also that the inflation rate is 1% and that the low- and high-altruism agents maintain money holding worth \$100 and \$200, respectively. Then the first type (low altruism) loses \$1 because of inflation but receives back \$1.5 from the government. The second type loses \$2 because of inflation and receives only \$1.5 from the government. Of course, all this analysis assumes, as does most of the literature mentioned in the introduction to which this paper compares, that the seignorage revenue is redistributed in a lump-sum manner.

Naturally, if the welfare weight of each group is positive (that is, $\partial W/\partial U_j > 0$, $j = H, L$), then $\gamma^*$ falls in between $\gamma_H^*$ and $\gamma_L^*$. Also, the second part of the proposition follows immediately from the fact that the low-altruism group achieves a lower level of utility. In fact, under a sufficiently high welfare weight on this group or a sufficiently high aversion towards inequality, $\gamma^* < 1$ (positive inflation rate). The following example demonstrates such a possibility.

*Example* Let the preference functions take the following forms: $V(c_j) = \ln(c_j)$, $L(m_j) = \ln(m_j) - \zeta m_j$ if $m_j < 1/\zeta$ and $\ln(1/\zeta) - 1$ if $m_j \geq (1/\zeta)$, $\zeta > 0$ (notice that $\bar{m} = 1/\zeta$). Let the production function be Cobb-Douglas, $f(k) = Ak^{\alpha}$, $\alpha \in (0, 1)$, and let the social welfare function be the one introduced by Atkinson (1970), $W(U_H, U_L) = [1/(1-\varepsilon)]\{q[(U_H)^{1-\varepsilon} - 1] + (1-q)[(U_L)^{1-\varepsilon} - 1]\}$, $\varepsilon \geq 0$) ($\varepsilon$ reflects society's aversion towards inequality; $\varepsilon = 0$ yields the Benthamite social welfare function, implying complete indifference to inequality, while as $\varepsilon \to \infty$, $W$ approaches the Rawlsian social welfare function, implying maximization of the utility of the worst-off group). Assume also the following parameter values: $\alpha = 1/3$, $\beta_H = 0.965$, $\beta_L = 0.900$, $\zeta = 0.003$ and $A = 10$.

Under this parameterization, $\gamma_L^*$ falls between 1.00 (when $q = 0$) and 0.983 (when $q = 0.999$). The corresponding inflation rates are 0% and 1.7%. On the other hand, $\gamma_H^* = R = 1.037$ (inflation rate $= -3.6\%$). Table 1 gives the values of $\gamma^*$ (top entry in each cell) obtained by varying the values of $q$ (the fraction of altruistic agents) and $\varepsilon$ (the distribution parameter). The corresponding inflation rates are also reported in the same table (the bottom entry in each cell). Notice that the optimal rate of inflation is positive for a significant range of parameter values. Also, the

**Table 1** The optimal inflation rate

| | $\varepsilon$ | | | |
|---|---|---|---|---|
| $q$ | **0** | **1.0** | **2.0** | **3.0** |
| 0 | 1.000<br>0.0 | 1.000<br>0.0 | 1.000<br>0.0 | 1.000<br>0.0 |
| 0.2 | 1.016<br>−1.6 | 0.999<br>0.1 | 0.990<br>1.0 | 0.987<br>1.3 |
| 0.4 | 1.022<br>−2.2 | 1.006<br>−0.6 | 0.994<br>0.6 | 0.987<br>1.3 |
| 0.6 | 1.027<br>−2.6 | 1.013<br>−1.3 | 0.999<br>0.1 | 0.990<br>1.0 |
| 0.8 | 1.030<br>−2.9 | 1.021<br>−2.1 | 1.008<br>−0.8 | 0.996<br>0.4 |
| 1.0 | 1.037<br>−3.6 | 1.037<br>−3.6 | 1.037<br>−3.6 | 1.037<br>−3.6 |

*Notes*: The top entry in each cell gives the optimal interest factor on money and the bottom entry gives the optimal inflation rate (in %).

higher the value of $\varepsilon$ (that is, the greater the aversion to extremes in the distribution), the closer $\gamma^*$ to $\gamma_L^*$ is (see Proposition 3).

## 4. Unequal transfers

So far I have assumed that the monetary transfers from the government are equal among all agents. This was intended to capture the case where each agent's type is private information. In this section I consider the case where the government makes unequal transfers to the two groups (I continue to assume, however, that transfers to members of the same group are equal). Let $z$ $(1-z)$ be the fraction of the newly issued money balances that is distributed, equally and in a lump-sum manner, among the high (low) altruism agents. Hence,

$$a_{Ht+1} = \frac{z}{q}(\mu - 1)\gamma_{t+1}\left[qm_{Ht} + (1-q)m_{Lt}\right]$$

$$a_{Lt+1} = \frac{1-z}{1-q}(\mu - 1)\gamma_{t+1}\left[qm_{Ht} + (1-q)m_{Lt}\right]$$

where it may be recalled that $(\mu - 1)\gamma_{t+1}[qm_{Ht} + (1-q)m_{Lt}]$ represents newly issued money balances (see eq. 8). In steady state, since $\mu\gamma = 1$,

$$a_H = \frac{z}{q}(1-\gamma)\left[qm_H + (1-q)m_L\right]$$

$$a_L = \frac{1-z}{1-q}(1-\gamma)\left[qm_H + (1-q)m_L\right]$$

Recall that the sign of $\gamma - 1$ determines whether the inflation rate is positive or negative; hence, it determines whether agents pay taxes (i.e. seigniorage) or receive positive monetary transfers. Furthermore, the sign of $z - q$ determines whether the high and the low altruism agents pay or receive more than their equal share. For example, if $\gamma < 1$ and $z > q$ then the high altruism agents receive more than their equal share, while the low altruism agents receive less than their equal share (since $1 - z < 1 - q$).

The constraints, one for each type, (16) in the government's problem (P) become:[14]

$$c_H + b_H = Rs_H + \gamma m_H + \frac{z}{q}(1-\gamma)\left[qm_H + (1-q)m_L\right] \tag{16H}$$

$$c_L + b_L = Rs_L + \gamma m_L + \frac{1-z}{1-q}(1-\gamma)\left[qm_H + (1-q)m_L\right] \tag{16L}$$

Note the following two special cases: (i) $q = z$, which is the case of equal transfers analysed before and (ii) $z = qm_H/[qm_H + (1-q)m_L]$, which is the case of proportional transfers, where $a_H = (1-\gamma)m_H$ and $a_L = (1-\gamma)m_L$; that is, the

[14] Note that the other first-order conditions for the utility maximization problem of each type do not change, since the transfers are made in a lump-sum manner and are taken as given by the agents.

transfers are proportional to each agent's money holdings. If there is only one type of agents, as is the case in most of the literature, then these two forms of distribution (proportional and equal) coincide and constitute the only form consistent with a constant money growth rate.

The government's problem (P) is now defined as: $\max_{\gamma} W = W(U_H, U_L)$ subject to (11)–(15), (16H), (16L), (17), and (18). Before we proceed to the characterization of the optimal money growth rate it should be noted that: (i) Proposition 1 still holds; that is, the capital stock is still invariant to changes in the money growth rate; (ii) Lemma 1 may not hold anymore; that is, if the monetary transfers to the low-altruism agents are high enough then their utility level may be higher than the one achieved by the high-altruism agents, despite the fact that the former type does not receive any bequests. Nevertheless, one can derive sufficient conditions that restore Lemma 1.

*Lemma 1′* If $\gamma \leq 1$ and $q \leq z$ or $\gamma > 1$ and $q > z$ then in steady-state (i) $c_H > c_L$ and $m_H \geq m_L$ (ii) $U_H \geq U_L$.

*Proof* The proof follows exactly the same steps as the proof of Lemma 1. □

If $\gamma < 1$ and $q < z$ then the government distributes money and the low altruism agents receive less than their equal share $(1-q > 1-z)$. Thus, the wealth inequality is maintained. The same is true if $\gamma > 1$ and $q > z$; the government collects money and the low altruism agents pay more than their equal share $(1-z > 1-q)$. Finally, if $\gamma = 1$ then there are no monetary transfers, while if $q = z$ (equal transfers/taxes) each group pays or receives its share and the high altruism agents continue to be wealthier because of the existence of positive bequests (see Lemma 1).

Next, I re-examine the results of Proposition 2 when a fraction $z \in [0, 1]$ of the the newly issued money balances is distributed to the high-altruism agents and the remaining $1-z$ is distributed to the low altruism agents.

*Proposition 2′* (i) The low-altruism agents prefer a positive, zero or negative inflation rate if their share of the newly issued money balances, $1-z$, is less, equal or greater than the fraction of the existing money balances that they hold $[(1-q)m_L/[qm_H + (1-q)m_L]]$. In other words,

$$\gamma_L^* \lesseqqgtr 1 \quad \text{if} \quad z \lesseqqgtr \frac{qm_H}{qm_H + (1-q)m_L}$$

(ii) The high-altruism agents prefer an inflation rate equal to (higher than) the Friedman-rule rate if their share of the newly issued money balances, $z$, is lower than or equal to (higher than) the fraction of the existing money balances that they hold $[qm_H/[qm_H + (1-q)m_L]]$. That is,

$$\gamma_H^* = R > 1 \quad \text{if} \quad z \leqq \frac{qm_H}{qm_H + (1-q)m_L} \quad \text{and} \quad \gamma_H^* < R \quad \text{otherwise.}$$

*Proof* The proof of each part follows, as before, from the maximization of $U(c_j, m_j)$, $j = H, L$, subject to eqs (11)–(15), (16H), (16L), (17), and (18). □

The previous intuition regarding $\gamma_L^*$ applies to this case as well. Namely, inflation changes the real income of the low-altruism agents by $(\gamma - 1)m_L$. At the same time, they receive a transfer from the government equal to $(1-\gamma)(1-z)[qm_H + (1-q)m_L]/(1-q)$. The net impact is $(1-\gamma)[q(1-z)m_H - (1-q)zm_L]/(1-q)$, which is positive if $\gamma < 1$ and $z < qm_H/[qm_H + (1-q)m_L]$ or $\gamma > 1$ and $z > qm_H/[qm_H + (1-q)m_L]$. The behavior of the high altruism agents is more subtle, since they face one more trade-off, that between their own and their children's consumption (bequests). This trade-off results in the modified golden rule. Thus, when $z$ is low then capital and money are complements for them, since following a decrease in $s_L$, which results from an increase in $\gamma$ they must save more in order to maintain the level of capital given by the modified golden rule. In equilibrium this results in an increase in their consumption, money balances, and bequests. Just as in the case of the infinite-horizon representative agent case or the case where there exist only high altruism agents, the optimal policy requires that the opportunity cost of money be set equal to zero, i.e. $\gamma_H^* = R$. If $z$ is high, on the other hand, then money and capital are not complements any more. The welfare effect from changes in their savings must balance the effect of the positive transfer from the government. In fact, $\gamma_L^*$ need not be lower than $\gamma_H^*$ anymore. For example, if $z = 1$, then $\gamma_L^* = R > \gamma_H^*$ (the complete picture regarding the optimal value of the inflation rate for each group, as $z$ runs from zero to one, is given in Fig. 1).

*Proposition 3′* (i) The value of the money growth rate that maximizes the social welfare function falls in between the rates preferred by the two groups, $\gamma_L^*$ and $\gamma_H^*$ (ii) The optimal value of $\gamma$ depends on the transfer parameter $z$.

*Proof* The proof of (i) follows trivially, as before, from the fact that $\partial W/\partial U_j > 0$, $j = H, L$. (ii) Given that $\gamma_L^*$ and $\gamma_H^*$ depend on $z$, so does $\gamma^*$. □

Obviously, there are infinite values of $\gamma^* \in [\min(\gamma_L^*, \gamma_H^*), R])$, one for each value of $z$, that maximize social welfare. Thus, additional notation is needed to denote the case of an optimal value of $\gamma$ for a particular value of $z$. Let $\gamma_j^*(\bar{z})$ denote the value of $\gamma$ preferred by group $j$ when $z = \bar{z}$; let also $z_j^*$ $j = H,L$ denote the value of $z$

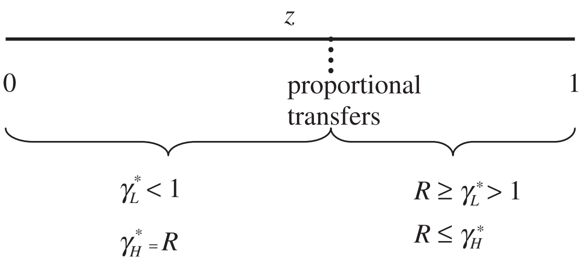


**Fig. 1.** Optimal inflation rate for each group

preferred by group $j$. The following proposition characterizes the pair of (z, $\gamma$(z)) that maximizes the utility level of each group.

*Proposition 4* (i) Among the infinite pairs of ($z$, $\gamma$(z)), where $z \in [0,1]$ and $\gamma \in (0, R]$, the low altruism agents prefer one of the following two pairs: either $(0,\ \gamma_L^*(0))$ or $(1, R)$. (ii) The high altruism agents prefer one of the two pairs: either $(0, R)$ or $(1,\ \gamma_H^*(1))$.

*Proof* (i) Simple differentiation yields $\partial U_L^*/\partial z < 0$ if $\gamma > 1$ and $\partial U_L^*/\partial z > 0$ if $\gamma > 1$. Hence, $z_L^* = 0$ if $\gamma < 1$ and $z_L^* = 1$ if $\gamma > 1$. Furthermore, $\gamma_L^*(1) = R$, since $\partial U_L^*/\partial\gamma|_{z=1,\,R=\gamma} > 0$. (ii) Similarly, $\partial U_H^*/\partial z > 0$ if $\gamma < 1$ and $\partial U_H^*/\partial z < 0$ if $\gamma > 1$. Thus, $z_H^* = 1$ if $\gamma < 1$ and $z_H^* = 0$ if $\gamma > 1$. Furthermore, it follows from Proposition 2$'$(ii) that $\gamma_H^*(0) = R$.

Each group prefers one of the following points on the ($z$, $\gamma$) plane: (i) the point where it receives all the newly issued money, or (ii) the point where it makes no payments and the money growth rate is at the Friedman-rule level; once again, in the latter case the money growth rate (inflation rate) is as low as possible and the opportunity cost of holding money is zero. Furthermore, there does not exist an unambiguous relationship between $U_L(0,\ \gamma_L^*(0))$ and $U_L(1, R)$. Indeed, suppose that the functions assume the same forms as in the Example above. Then, depending on the value of $q$ when $z = 0$, $U_L$ may be higher or lower than the value of $U_L$ when $z = 1$ (the latter is independent of $q$). More specifically, assuming that $\beta_H = 0.965$, $\beta_L = 0.900$, and $\zeta = 0.003$ then $U_L(z = 0,\ \gamma_L^*(0) = 0.895,\ q = 0.8) = 77.968 > U_L(z = 1,\ \gamma_L^* = R) = 73.263 > U_L(z = 0,\ \gamma_L^*(0) = 0.941,\ q = 0.5) = 69.175$. Similarly, the high altruism agents do not always prefer one of the two points $(1,\ \gamma_H^*(1))$ and $(0,\ \gamma_H^*(0)) = (0,\ R)$; e.g., with $\beta_H = 0.965$, $\beta_L = 0.900$ and $\zeta = 0.01$, $U_H(z = 1,\ \gamma_H^*(1) = 0.85,\ q = 0.2) = 196.649 < U_H(z = 0,\ \gamma_H^*(0) = R,\ q = 0.2) = 197.039$, but $U_H(z = 1,\ \gamma_H^*(1) = 0.746,\ q = 0.1) = 214.957 > U_H(z = 0,\ \gamma_H^*(0) = R,\ q = 0.1) = 209.962$. Finally, notice that the optimal value of $\gamma$ that corresponds to the optimal value of $z$, $\gamma^*(z^*)$, may still be lower than one.

## 5. Conclusions

This paper has developed a relatively simple framework to analyze the role of monetary policy in an economy with two assets, capital and money, as well as two types of agents who differ with respect to their degree of altruism towards their heirs. It therefore differs from the standard monetary growth models in that it dispenses with the assumption that the economy is populated by a representative agent in each generation.

First, it has been shown that changes in the money growth rate can have significant distributional effects. Second, the optimal rate of monetary expansion is in general higher than the one implied by the Friedman rule and may in fact result in a small but positive rate of inflation, despite the fact that the capital stock is invariant to changes in the money growth rate. In other words, it has been shown that the invariance of the capital stock to changes in the money growth rate is not sufficient for the Friedman rule (it is also not necessary because, as mentioned in the introduction, in models where there exists a reverse Tobin effect the Friedman rule may still be optimal). Finally, the optimal money growth rate increases as the society's aversion towards inequality increases.

## Acknowledgements

I would like to thank the Editor James Forder and two anonymous referees for helpful comments and suggestions on earlier versions of the paper.

## References

**Abel, A.B.** (1985) Dynamic behavior of capital accumulation in a cash-in-advance model, *Journal of Monetary Economics*, **16,** 55–71.

**Atkinson, A.B.** (1970) On the measurement of inequality', *Journal of Economic Theory*, **2,** 244–63.

**Bhattacharya, J. and Haslag, J.H.** (2001) On the use of the inflation tax when nondistortionary taxes are available, *Review of Economic Dynamics*, **4,** 823–41.

**Brock, W.A.** (1974) Money and growth: the case of long run perfect foresight, *International Economic Review*, **15,** 750–77.

**Brock, W.A.** (1990) Overlapping generations models with money and transactions costs, in B.M. Friedman and F.H. Hahn (eds), *Handbook of Monetary Economics*, North-Holland, Amsterdam.

**Chari, V.V., Christiano, L.J., and Kehoe, P.J.** (1996) Optimality of the Friedman rule in economies with distortionary taxes, *Journal of Monetary Economics*, **37,** 203–23.

**Chari, V.V. and Kehoe, P.J.** (1999) Optimal fiscal and monetary policy, in J.B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics*, Elsevier, Amsterdam.

**Correia, I. and Teles, P.** (1996) Is the Friedman rule optimal when money is an intermediate good?, *Journal of Monetary Economics*, **38,** 223–44.

**Correia, I. and Teles, P.** (1999) The optimal inflation tax, *Review of Economic Dynamics*, **2,** 325–46.

**Drazen, A.** (1981) Inflation and capital accumulation under a finite horizon, *Journal of Monetary Economics*, **8,** 247–60.

**Freeman, S.** (1993) Resolving differences over the optimal quantity of money, *Journal of Money, Credit, and Banking*, **25,** 801–11.

**Friedman, M.** (1969) *The Optimum Quantity of Money and Other Essays*, Aldine, Chicago.

**Ireland, P.N.** (2003) Implementing the Friedman rule, *Review of Economic Dynamics*, **6,** 120–34.

**Kim, K.** (2001) Optimal quantity of money with two-sided altruism, *Seoul Journal of Economics*, **14,** forthcoming.

**Levine, D.K.** (1991) Asset trading mechanisms and expansionary policy, *Journal of Economic Theory*, **54,** 148–64.

**Mulligan, C.B. and Sala-i-Martin, X.** (1997) The optimum quantity of money: theory and evidence, *Journal of Money, Credit, and Banking*, **29,** 687–715.

**Sidrauski, M.** (1967) Rational choice and patterns of growth in a monetary economy, *American Economic Review*, **57,** 534–44.

**Shi, S.** (1999) Money, capital, and redistributive effects of monetary policies, *Journal of Economic Dynamics and Control*, **23,** 565–90.

**Smith, R.T.** (1998) The Friedman rule and optimal monetary policy, *Canadian Journal of Economics*, **31,** 295–302.

**Weiss, L.** (1980) The effects of money supply on economic welfare in the steady state, *Econometrica*, **48,** 565–76.

**Woodford, M.** (1990) The optimum quantity of money, in B.M. Friedman and F.H. Hahn (eds), *Handbook of Monetary Economics*, North-Holland, Amsterdam.