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Government Expenditure Financing in an Endogenous Growth Model: A Comparison

RECENTLY THERE HAS BEEN a renewed interest in issues of economic growth. The primary contribution of the new literature, pioneered by Romer (1986), Lucas (1988), and Rebelo (1991), has been to endogenize the growth rate of the economy and, in so doing, to promote our insights on the sorts of economic policies that can enhance the growth performance. One strand of the endogenous growth literature, in particular, examines the role of fiscal policy (especially taxation) in the growth process (see, for example, Barro 1990, King and Rebelo 1990, Alogoskoufis and Ploeg 1991, and Rebelo 1991) while another tries to identify mechanisms through which changes in monetary policy might influence economic growth (see Mino 1991, Ploeg and Alogoskoufis 1994, Jones and Manuelli 1993, and Wang and Yip 1993).

Drawing on both strands, this paper attempts to answer the following question. How should a government finance an expenditure path? Should it use fiscal (raise taxes) or monetary methods (issue money)? The answer to this question is crucial since any government considering existing or new spending programs must decide on how to raise the necessary revenue. It is well understood in the public finance/macroeconomics literature that different government financing methods may have different effects on the aggregate economy (see, for example, Haliassos and Tobin 1990). Within the endogenous growth paradigm the issue becomes even more im-

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portant since various economic policies have different impacts not only on the level of output but also on its growth rate.[1]

Turnovsky (1992) examines the impact of different methods of government expenditure financing in a Ramsey-Cass economy. Ploeg and Alogoskoufis (1994), on the other hand, develop an endogenous growth model with noninterconnected overlapping generations to examine the effects on growth and inflation of lump-sum-tax-financed, debt-financed, and money-financed increases in government consumption. Finally, Cooley and Hansen (1992) calibrate a real business cycle model and estimate the benefits of replacing a capital tax by other forms of taxation.

In this paper, we utilize a simple monetary endogenous growth framework to assess the *relative* merits of alternative modes of expenditure financing. We emphasize the transactional role of money via a generalized cash-in-advance or liquidity constraint. More specifically, we require that all consumption purchases but, different from Stockman (1981), only a fraction of investment purchases be made using cash. As long as that fraction is positive, money has real effects. In particular, an increase in the money growth rate will decrease the expansion rate of the economy.[2]

Following the differential tax incidence approach, we compare the growth and welfare effects of a permanent increase in government expenditure-income ratio under two alternative modes of financing: an increase in the income tax rate and an increase in the nominal money growth rate (seigniorage or inflation tax). The policy experiment we conduct is as follows. The government tries to maintain a continuously balanced budget or equivalently a constant share of government expenditure in GNP.[3] In particular, in each of the two financing policies the corresponding tax rate is endogenously determined to finance the given expenditure/income ratio, while the other tax rate is set to zero to facilitate comparison. Unlike Barro (1990), the services provided by the government are assumed not to enter households' utility or production functions. Although this affects the level of the growth rate and hence of welfare, it does not affect the *relative* ranking of alternative financing methods.

First, we derive some results regarding the growth and inflation rates under alternative financing schemes. Not only are these results interesting in their own right, but they are also used in the welfare analysis of different policies. We find, not surprisingly, that an increase in seigniorage or in the income tax rate decreases the growth rate of the economy. This is due to the fact that both policies decrease the net rate of return to investment. Nevertheless, we are able to show that, for any given government size, the decrease in the growth rate is less under money financing than

1. It is well understood by now that "even small growth effects can swamp large increases in levels. [That is why] the difference between exogenous and endogenous growth is of more than academic interest" (Romer 1989, pp. 95–96).

2. This adverse effect of money growth on output growth is also present in the cash-in-advance models adopted in Jones and Manuelli (1993) and in Mino (1991) as well as in the shopping-time model utilized in Wang and Yip (1993). Nevertheless, their focus is on assessing the effects of monetary policy and/or inflation while we are interested in comparing the effects of different financing schemes on economic growth and welfare.

3. As in Barro (1990), if the government does not maintain a constant share of its expenditure in income then the economy will not follow a balanced growth path.

under income tax financing.[4] Furthermore, we compare the inflation rates pertaining to each regime. Interestingly, we find that the inflation rate achieved under money financing is higher than the one achieved under income tax financing, pointing to a trade-off between growth and inflation. We also examine the mix of these two methods of financing that maximizes the growth rate and find that this should involve only seigniorage and not income taxation.[5] Finally, we investigate the welfare effects of the two financing schemes and find that the mix that maximizes the representative household's welfare depends crucially on the fraction of investment purchases that are subject to the cash-in-advance constraint.

The remainder of the paper is organized as follows. Section 1 describes the economy and Section 2 analyzes its equilibrium path under money and income tax financing. Section 3 compares the effects of these two budgetary policies on economic growth and on inflation while section 4 examines the growth-maximizing tax structure. Section 5, on the other hand, determines the financing scheme that maximizes the welfare of the representative household. Finally, section 6 summarizes our conclusions and discusses some future directions for further research.

1. THE ECONOMY

A. The Environment

Consider an economy consisting of a large (but finite) number of homogeneous agents (households) who are infinitely lived and have perfect foresight. Their instantaneous utility function depends on per capita consumption (c) and takes the constant elasticity of intertemporal substitution (CEIS) form, that is, $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$, $\sigma > 0$.[6]

Each household also has access to a common technology described by $y(t) = f(k(t))$, where y and k denote output and capital per worker, respectively. In the absence of exogenous technical progress, the existence of a nondegenerate balanced

4. Using calibration methods, Cooley and Hansen (1992) arrive at a similar conclusion for the U.S. economy in a cash-in-advance real business cycle model. More specifically, they find that the replacement of a labor-income tax or a capital-income tax by an inflation tax increases the *level* of capital stock.

5. The importance of seigniorage as a revenue source has been realized among economists as well as among policy makers. For example, six out of the seven hyperinflating countries examined in Cagan (1956) were able to procure between 3 and 15 percent of national income by issuing money. During the period 1960–78, among twenty-four countries, the average share of seigniorage to GNP was greater than 2 percent. During the same period, in thirteen countries, more than 10 percent of government revenue came from seigniorage (see Fischer 1982). Finally, in a recent paper, Dornbusch and Fischer (1991) conclude that "seigniorage revenues account for a significant share of government revenues in most of the moderate inflation countries." For example, in Greece and Portugal, over the period 1982–87, seigniorage accounted for 11.2 percent and 6.5 percent, respectively, of government revenues.

6. The elasticity of intertemporal substitution in consumption is given by $1/\sigma > 0$. Applying L' Hospital's rule, one can find that $\sigma = 1$ corresponds to the logarithmic utility function. It can also be shown that the CEIS functional form is required for the existence of a balanced growth equilibrium path (defined below), which, in the case of a growing economy, is the appropriate analogue to a steady state (see King, Plosser, and Rebelo 1988). In fact, given a linearly homogeneous production function, (asymptotically) constant rate of time preference and (asymptotically) constant elasticity of intertemporal substitution are not only necessary but also sufficient conditions for the existence of a(n) (asymptotically) balanced growth equilibrium path (see Palivos, Wang, and Zhang 1994).

growth equilibrium (to be defined below) requires that the production function exhibits constant returns to scale with respect to per capita capital. Following Rebelo (1991), we assume $y(t) = Ak(t)$, where $A \in (0, \infty)$ denotes the (constant) marginal product of capital.[7] Finally, without any loss of generality, we assume that there is no population growth and no depreciation of the capital stock.

At each instant of time, the representative household faces two constraints. The first is a modified Sidrauski-type budget constraint

$$c(t) + \dot{k}(t) + \dot{m}(t) = (1 - \tau)Ak(t) - \pi(t)m(t) \tag{1}$$

where $m(t)$, τ, and $\pi(t)$ denote real money balances, a proportional income tax rate, and the inflation rate, respectively, and $\dot{x} \equiv dx/dt$ for any variable x. The second constraint is a generalized cash-in-advance (CIA) or liquidity constraint;[8] namely, all purchases of current consumption and a fraction, ϕ, of investment must be made using cash, that is,[9]

$$c(t) + \phi\dot{k}(t) \leq m(t) \ . \tag{2}$$

A government operates in the following way. There is a given sequence of government spending $\{g(t)\}_{t=0}^{\infty}$, denominated in units of the period t consumption good as well. The government raises revenue by imposing a proportional income tax and/or by printing money, that is, by imposing an inflation tax. All revenue then raised in period t is used to finance government purchases; hence, the government budget constraint is[10]

$$g(t) = \tau y(t) + \mu m(t) \tag{3}$$

where μ is the (constant) money growth rate.[11]

B. Equilibrium Path

Consider first the representative agent's optimization problem. Let $\rho \in (0, \infty)$ denote the agent's subjective rate of time preference, who then seeks to maximize

7. The only linearly homogeneous function of one argument is the linear function.

8. For an overview of various monetary growth models, see Dornbusch and Frenkel (1973) and Wang and Yip (1992).

9. The cases $\phi = 0$ and $\phi = 1$ are associated with Lucas (1980) and Stockman (1981). For an analysis of various monetary effects in the intermediate case where $0 < \phi < 1$, see Koenig (1987 and 1989) and Palivos, Wang, and Zhang (1993). One could also allow for a fraction of the consumption purchases to be subject to the liquidity constraint. This, however, would complicate the model without changing the results.

10. The real revenue raised from seigniorage is $\dot{M}/P = (\dot{M}/M)\,(M/P) = \mu m$, where M and P denote nominal money balances and the price level, respectively.

11. Constant rates of income taxation and money growth are required for the existence of a balanced growth equilibrium path.

$$U = \int_0^{\infty} u(c(t))\, e^{-\rho t}\, dt \ , \tag{4}$$

subject to (1) and (2), taking the initial capital stock $k(0)$, and the paths of inflation and money supply as given. The current-value Lagrangian is defined by

$$L(k, m, c, z, \lambda_1, \lambda_2, \lambda_3, t) \equiv (c^{1-\sigma} - 1)/(1 - \sigma) + \lambda_1[(1 - \tau)Ak - \pi m - c - z] + \lambda_2(m - c - \phi z) + \lambda_3 z \ ,$$

where λ_1 and λ_3 denote costate variables, λ_2 is a Lagrange multiplier, and z is a slack variable ($z \equiv \dot{k}$). We have also dropped the time index t to simplify the notation. Applying Pontryagin's Maximum Principle, we find that the optimizing program is described by the following first-order conditions:

$$c^{-\sigma} - \lambda_1 - \lambda_2 = 0 \ , \tag{5}$$

$$-\lambda_1 - \phi\lambda_2 + \lambda_3 = 0 \ , \tag{6}$$

$$\dot{\lambda}_1 = \rho\lambda_1 + \lambda_1\pi - \lambda_2 \ , \tag{7}$$

$$\dot{\lambda}_3 = \rho\lambda_3 - \lambda_1(1 - \tau)A \ , \tag{8}$$

$$\lambda_2 \geq 0, (m - c - \phi z) \geq 0, \lambda_2(m - c - \phi z) = 0 \ , \tag{9}$$

together with the private budget constraint, (1), and the transversality conditions:

$$\lim_{t \to \infty} e^{-\rho t}\lambda_1(t)m(t) = \lim_{t \to \infty} e^{-\rho t}\lambda_3(t)k(t) = 0 \ .$$

Moreover, these conditions together with the government budget constraint, (3), and the money market equilibrium condition, $\dot{m}(t) = [\mu - \pi(t)]m(t)$, determine the equilibrium allocation in this economy. It can be shown that, similarly to other one-sector models (for example, Barro 1990 and Rebelo 1991), the initial consumption and real money balances jump so that the balanced growth path is obtained instantaneously; that is, there are no transitional dynamics in this economy.[12] We therefore concentrate exclusively on the balanced growth equilibrium path, which is defined as the path along which $c(t)$, $k(t)$, $m(t)$, $\lambda_1(t)$, $\lambda_2(t)$, and $\lambda_3(t)$ grow at constant rates and is the appropriate analogue to a steady state (see, for example, Lucas 1988). We also concentrate on the case where $\lambda_2 > 0$ and the CIA constraint is binding; that is, $m = c + \phi\dot{k}$.[13]

12. The proof is provided in a supplement available from the authors upon request.
13. The case where $\lambda_2 = 0$ and $c + \phi\dot{k} < m$ is examined in a later section.

Equation (7) can be rewritten as

$$\lambda_2/\lambda_1 = \rho + \pi - \dot{\lambda}_1/\lambda_1 , \tag{10}$$

which implies that λ_2/λ_1 is constant, that is, λ_1 and λ_2 grow at the same rate.

Next we rewrite (5) as $c^{-\sigma} = \lambda_1(1 + \lambda_2/\lambda_1)$. Taking logarithms of both sides and then differentiating with respect to time, we obtain

$$\dot{\lambda}_1/\lambda_1 = -\sigma\theta . \tag{11}$$

where $\theta \equiv \dot{c}/c$ denotes the growth rate of per capita consumption.

Furthermore, equations (1), (2), and (3) imply that consumption, real money balances, capital stock, output, and government spending grow at the same rate:

$$\dot{c}/c = \dot{m}/m = \dot{k}/k = \dot{y}/y = \dot{g}/g = \theta . \tag{12}$$

Finally, the money market equilibrium condition implies that $\dot{m}/m = \mu - \pi$ or $\pi = \mu - \theta$.

Note that $\rho + \sigma(\dot{c}/c) = \rho + \sigma\theta$ is the rate of return on consumption (real interest rate). Hence, $R \equiv \rho + \sigma\theta + \pi = \rho + (\sigma - 1)\theta + \mu$ is simply the nominal interest rate. Furthermore, combining (6), (8), (10), and (11), one obtains

$$\rho + \sigma\theta = (1 - \tau)A/(1 + \phi R) , \tag{13}$$

which is a modified Keynes-Ramsey rule determining capital accumulation. It requires that the marginal rate of intertemporal substitution be equal to the marginal rate of transformation or that the real interest rate be equal to the after-tax return on investment. In fact, if $\phi = 0$ then (13) becomes $\rho + \sigma\theta = (1 - \tau)A$, which is the standard Keynes-Ramsey rule in an economy with no depreciation and no population growth. [A further discussion on (13) is provided below, where the two modes of financing are considered separately; see also equation (14) in Koenig (1987) and the accompanying discussion.]

We restrict our analysis to the case where an equilibrium exists and the growth rate of the economy is positive. It can be shown (see Sala-i-Martin 1990 and Barro 1990) that, in order for these two presumptions to be satisfied in a barter economy with linear technology, it is required that $A > \rho > A(1 - \sigma)$. Given $A > 0$ and $\rho > 0$, $A > \rho$ and $\sigma \geq 1$ are then sufficient conditions for all parts of the inequality to hold. Since a barter equilibrium is a special case within our monetary framework and in order to simplify the analysis, we henceforth assume that these sufficient conditions are always satisfied.[14] Finally, notice that in general to ensure bounded lifetime utility and hence the existence of a maximum, one needs to assume that $\rho >$

14. The assumption $\sigma \geq 1$, in particular, is supported by empirical evidence found in Weber (1975) and Hall (1988) which indicates that the intertemporal elasticity of substitution $(1/\sigma)$ is significantly less than one.

$(1 - \sigma)\theta$. Given our assumptions on ρ, σ, and θ, this condition is automatically satisfied.

2. ALTERNATIVE METHODS OF FINANCING PUBLIC SPENDING

This section examines the effects of the two different financing methods, mentioned above, on the rate of economic growth. Let θ_T and θ_M denote the growth rates under an income tax, and an inflation tax, respectively.

A. Income Tax Financing

Under income tax financing, $\mu = 0$, $g = \tau y$, and (13) becomes

$$F(\theta_T) = A\gamma \ . \tag{14}$$

where $F(\theta) \equiv \phi\sigma(1 - \sigma)\theta^2 - \{(\sigma - 1)\phi\rho + \sigma(1 + \phi\rho)\}\theta + A - \rho - \phi\rho^2$, and $\gamma \equiv g/y$ denotes the government size.

Before examining the effect of a change in γ and hence in τ, we show that there exists only one positive growth rate θ_T in the neighborhood of which we can conduct comparative statics exercises. First, note that (14) is a quadratic equation in θ_T. We claim that the constant term must be positive. The easiest way to see this is to think of time as being discrete and to consider a decrease in investment in period t by one unit. This frees up ϕ units of real money balances which can be used to finance consumption in period t while the remainder $(1 - \phi)$ can be held in the form of real balances to finance consumption in period $t + 1$. The increase in lifetime welfare resulting from this action equals $\{\phi + (1 - \phi)/(1 + \rho)\}u_c = \{(1 + \phi\rho + \rho + \phi\rho^2)/(1 + \rho)^2\}u_c$ where u_c indicates the marginal utility of consumption. The loss in utility, on the other hand, due to foregone consumption in period $t + 2$ that the proceeds of investment would have purchased is equal to $\{[1 + (1 - \gamma)A]/(1 + \rho)^2\}u_c$. In the absence of inflation and growth in consumption the cost should equal the benefit and hence the constant term in (14) should be non-negative. Inflation and positive growth then make current consumption more attractive and hence $(1 - \gamma)A$ must exceed $\rho + \phi\rho^2$ by even more $[(1 - \gamma)A - \rho - \phi\rho^2 > \phi\rho \geq 0]$. Note, for future reference, that this condition imposes an upper bound on the government size; namely, $0 < \gamma < \gamma_1 \equiv (A - \rho - \phi\rho^2)/(A - \rho - \phi\rho^2 + \phi\rho) < 1$.

Furthermore, the condition $\sigma \geq 1$ implies that the coefficient of θ_T^2 in (14) is non-positive. The discriminant of the quadratic equation, on the other hand, is positive. One can therefore conclude that there exists a unique positive growth rate.

The effect of an increase in γ, and hence in τ, on θ_T is given by

$$d\theta_T/d\gamma = -A/D_T < 0 \ , \tag{15}$$

where $D_T \equiv \sigma + \phi\{\sigma[\rho + (\sigma - 1)\theta_T] + (\sigma - 1)(\rho + \sigma\theta_T)\} > 0$. There are two negative effects on the rate of economic growth compared to a frictionless world.

First, as indicated by (13), the existence of the CIA constraint imposes a negative effect on investment, since it decreases the rate of return on investment and acts in effect as a tax on capital. Second, an increase in the flat income tax rate, τ, decreases the net marginal product of capital and hence suppresses economic growth (reduces θ) even further.

B. Money Financing

Next consider the case where public spending is financed by an inflation tax (seigniorage); hence $\tau = 0$ and $\mu m = g$.[15] We can thus write $\mu m/k = A\gamma$. Combining these equations with (1) and (2), we obtain the following expression for the (endogenous) money growth rate: $\mu = \gamma/[(1 - \gamma) - (1 - \phi)(\theta_M/A)]$, and by substituting μ away in (13)

$$F(\theta_M) = G(\theta_M) \ , \tag{16}$$

where $G(\theta) \equiv A\gamma\phi(\rho + \sigma\theta)/[(1 - \gamma)A - (1 - \phi)\theta]$. Simple differentiation shows that $F' < 0$ for $\theta \geq 0$, $F'' < 0$, and $G' > 0$. Furthermore $F(\infty) = -\infty$. The two functions are depicted in Figure 1. The existence of a positive equilibrium growth rate requires that $F(0) > G(0)$, which is satisfied if $\gamma < \gamma_1$. [The uniqueness of a positive growth rate satisfying (16) is then easily established (see Figure 1).] Furthermore, it follows from the resource constraint that $c(t) > 0$ requires that $(1 - \gamma)A > \theta_M$. Since, as shown below, the maximum value of θ_M is $(A - \rho)/\sigma$, a sufficient condition for this is $0 < \gamma < \gamma_2 \equiv 1 - [(A - \rho)/\sigma A]$. We henceforth assume that

$$\gamma < min\ (\gamma_1, \gamma_2) \ . \tag{E}$$

Differentiation of (16) yields the effect of an increase in the government size on the rate of economic growth

$$d\theta_M/d\gamma = -(A/D_M)\ \phi(\rho + \sigma\theta_M)(1 + \mu)\ (k/m) < 0 \ , \tag{17}$$

where $D_M \equiv \sigma + \phi\{\sigma[\rho + (\sigma - 1)\theta_M + \mu] + (\sigma - 1)(\rho + \sigma\theta_M)\} + (\rho + \sigma\theta_M)(1 - \phi)(\phi\mu k/m) > 0$. An increase in the government size requires a higher money growth to finance it. In addition to the negative effect due to the presence of the CIA constraint, a higher money growth rate increases the inflation rate, which erodes the purchasing power of money, makes the CIA constraint more restrictive to investment goods, and hence is detrimental to growth. The magnitude of both (negative) effects is proportional to the share parameter, ϕ. In particular, if the CIA constraint applies only to consumption purchases, that is, if $\phi = 0$, then both effects disap-

15. The actual tax that inflation imposes on money holders is the loss in the value of their real balances, πm. Traditionally, inflation tax and seigniorage are equal in steady state where there is no output growth and $\mu = \pi$. In our framework, however, where there is perpetual growth, the difference between government revenue and inflation tax is $\mu m - \pi m = \theta m = \dot{m}$, which increases over time.

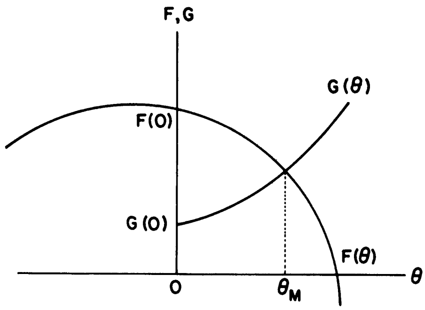

FIG. 1. Existence of θ_M under Money Finance

pear, $(d\theta_M/d\gamma) = 0$, and the money growth rate is superneutral *in the growth rate sense*; changes in μ do not affect θ (the *growth rate* of output and capital). A similar result is found in Stockman (1981) and Abel (1985); namely, if $\phi = 0$ then money is superneutral, that is, changes in μ do not affect the *levels* of capital and output.[16] Intuitively, if cash is not required to purchase capital then changes in the money growth rate and hence in the inflation rate do not affect the cost of investment.

3. COMPARISON OF GROWTH AND INFLATION RATES

This section compares the paths of output growth and inflation resulting from the two financing methods first with the paths obtained in a first-best environment and then with each other.

A. *First-Best Analysis*

Consider first the growth rate of output achieved in the absence of any liquidity constraints on investment as well as of distortionary taxation (θ^*). This is given by $(A - \rho)/\sigma$ as it can be found by substituting $\phi = \tau = 0$ in (13). Notice that $d\theta^*/d\gamma = 0 \geq \max(d\theta_T/d\gamma,\ d\theta_M/d\gamma)$. Furthermore, $\theta^* \geq \theta_T = \theta_M$ for $\tau = \mu = \gamma = 0$. Thus, $\theta^* \geq \max(\theta_T, \theta_M)$.

Achieving this first-best growth rate, θ^*, given the need to finance a sequence of government spending and the CIA constraint with $\phi > 0$, requires two sets of policy measures. First, since lump-sum taxation does not affect the rate of return to investment and hence does not hinder capital accumulation, the government should use it exclusively as a financing method. Second, the opportunity cost of holding money (nominal interest rate) should also be eliminated, destroying thus the motivation for economizing on money holdings. To succeed in this, the monetary authority should

16. Abel (1985) shows that unanticipated changes in the money growth rate do not affect the level of capital along the transition path as well. Anticipated changes, however, alter the time-paths of consumption and investment (see Koenig 1987).

set the money growth rate equal to $\mu^* = -[\rho + (\sigma - 1)\theta^*]$, in which case $\pi^* = \mu^* - \theta^* = -A$; that is, μ^* equates the rate of return to money ($-\pi$) with the rate of return to capital (A). This result is known in the literature as the Friedman rule (Friedman 1969) and has been derived in surprisingly different models of exogenous growth (see Woodford 1990).[17] Finally, using (13), one can show that the growth rate under these two policy measures, lump-sum taxation and $\mu = \mu^*$, $\theta_L|_{\mu=\mu^*}$, will equal θ^*.[18]

As it is shown formally below, a fall in the money growth rate decreases inflation and promotes capital accumulation. Hence, one may think that decreasing μ below μ^* can promote output growth even further. Nevertheless, μ^* is the lowest bound for μ since if $\mu < \mu^*$ then an equilibrium growth path does not exist (see the Claim in the Appendix).

B. Comparing the Growth Rates

From (15) and (17), we see that both income taxation and seigniorage reduce the economic growth rate. Nevertheless, we show that, given any government size, money financing is less distortionary to economic growth than income tax financing. More formally, we prove the following proposition.

PROPOSITION 1: *For any $\gamma > 0$, which satisfies the existence condition (E), $\theta_M > \theta_T$.*

PROOF: Consider first the case where the cash-in-advance constraint applies only to consumption purchases and thus $\phi = 0$. As shown above, in this case (17) implies that $d\theta_M/d\gamma = 0$. Using (15) and (17) we can write

$$d\theta_M/d\gamma = 0 > d\theta_T/d\gamma = -A/\sigma, \qquad \text{for } \gamma > 0 .$$

Furthermore, if $\gamma = 0$ then $\theta_M(0) = \theta_T(0)$. Hence $\theta_M > \theta_T$ for any $\gamma > 0$.

For $0 < \phi \leq 1$, define $\tilde{\theta}$ such that $G(\tilde{\theta}) = A\gamma$.[19] Using the definition of $G(\cdot)$, $G(\theta) \equiv [A\gamma\phi(\rho + \sigma\theta)]/[(1 - \gamma)A - (1 - \phi)\theta]$, it is easy to show that

$$\tilde{\theta} = [(1 - \gamma)A - \phi\rho]/(1 - \phi + \phi\sigma) .$$

Simple differentiation then yields $d\tilde{\theta}/d\phi = -[\rho + (\sigma - 1)(1 - \gamma)A]/(1 - \phi + \phi\sigma)^2 < 0$. Similarly, using (14), $d\theta_T/d\phi < 0$. Notice also that $\tilde{\theta}|_{\phi=1} = \theta_T|_{\phi=0} = [(1 - \gamma)A - \rho]/\sigma$. Hence, given ϕ, $\tilde{\theta} > \theta_T$ (see Figure 2 for a graphical representation of

17. For a detailed analysis of the Friedman Rule in a second-best environment, see also Phelps (1973), Helpman and Sadka (1979), Turnovsky and Brock (1980), Kimbrough (1986), and the literature cited therein.

18. Using (10) and (11) we can show that if $\mu = \mu^*$, $\lambda_2 = 0$ and thus the cash-in-advance constraint is not binding. In the absence then of any distortionary tax, for example, an income tax, the economy will grow at the first-best rate θ^*.

19. Condition (E) is sufficient for the existence of $\tilde{\theta} > 0$.

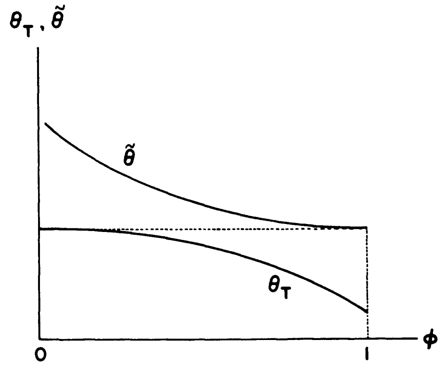

FIG. 2. $\tilde{\theta}$ versus θ_T

the relationship between θ and θ_T). Applying Lemma 1 in the Appendix the result follows.■

It is easy to grasp the intuition by adopting a graphical approach. Under $\phi = 0$, cash is not required to purchase capital and hence the cost of investment is unaffected by changes in the money growth rate and/or the inflation rate. Since income tax financing is growth suppressing, seigniorage is a superior method of financing. The situation is depicted in Figure 3.

As ϕ increases from 0 to 1, $F(\theta)$ shifts down. Similarly, $G(\theta)$ shifts up and becomes upward sloping since seigniorage is now distortionary. This then reduces both θ_M and θ_T. Nevertheless, the downward shift of $F(\theta)$ always dominates the upward shift of $G(\theta)$, implying that the direct effect of income taxation on growth is more distortionary than the effect of the inflation tax through the cash-in-advance constraint. The general case is presented in Figure 4.[20]

Finally, we adopt specific numerical values for the various parameters of the model in order to provide some estimates regarding the difference in the growth rates. More specifically, we postulate that $\sigma = 2$, $\rho = 0.03$, $A = 0.1$, $\gamma = 0.35$.[21] Table 1 shows that the relative change in the growth rate when switching from one regime to another can be very significant.

C. Comparing the Inflation Rates

Let $\pi_L|_{\mu=\mu^*}$, π_T, and π_M denote the inflation rates under a lump-sum tax accompanied by the optimal quantity of money, an income tax, and an inflation tax, re-

20. According to Koenig (1987), unpublished flow-of-funds data from the Federal Reserve indicate that checkable deposits of households are about three times as large as those of the nonfinancial business sector. This suggests the value 1/3 for ϕ.

21. For example, in 1987 government outlays as a percentage of GDP were 49.4 in France, 44.4 in Germany, 62.7 in Sweden, and 37.9 in the United Kingdom. The same year in the United States they were 34.9 percent of GNP (National Accounts of OECD Countries 1974–1987, volume 2).

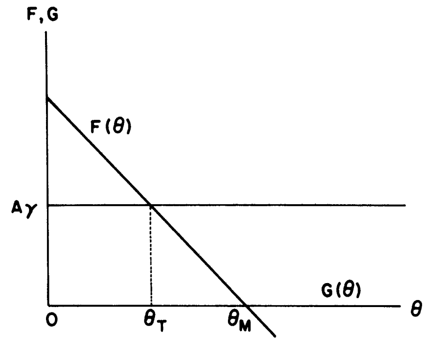

FIG. 3. Comparison between the Growth Rates when $\phi = 0$

spectively. First, we show that given any $\gamma > 0$, $\pi_L|_{\mu=\mu^*}$ is the lowest. Recall that $\pi_i = \mu_i - \theta_i$, where $i = L|_{\mu=\mu^*}$, T, M, and $\mu^* = -[\rho + (\sigma - 1)\theta_L|_{\mu=\mu^*}] < 0$, $\mu_T = 0$, and $\mu_M > 0$. Hence, using various parts of the inequality $A > (A - \rho)/\sigma = \theta_L|_{\mu=\mu^*} > \max(\theta_T, \theta_M)$, we obtain

$$\pi_L|_{\mu=\mu^*} = \mu^* - \theta^* = -A < -\theta_T = \pi_T ,$$

and

$$\pi_L|_{\mu=\mu^*} = \mu^* - \theta^* < \mu_M - \theta_M = \pi_M .$$

Second, we compare the inflation rates under the two distortionary tax schemes. Although $-\theta_M < -\theta_T$, $\mu_M > 0 = \mu_T$, and hence in general $\pi_M \gtreqless \pi_T$. Nevertheless, if the real interest rate is less than or equal to 100 percent ($A \leq 1$), one can show that $\pi_M > \pi_T$.

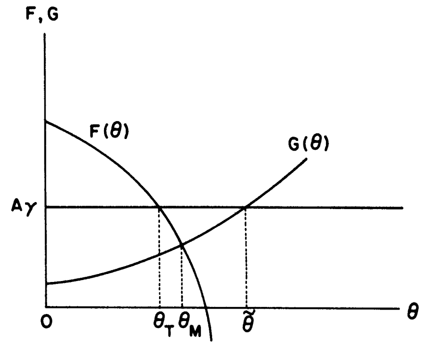

FIG. 4. Comparison between the Growth Rates when $\phi \in (0,1]$

TABLE 1

COMPARISON OF GROWTH RATES

	ϕ 0	1/3	2/3	1
$\theta_L \mid_{\mu=\mu^*}$	0.0350	0.0350	0.0350	0.0350
θ_M	0.0350	0.0247	0.0199	0.0165
θ_T	0.0175	0.0170	0.0165	0.0161

NOTES: Parameter values: $\gamma = 0.35$, $\sigma = 2$, $\rho = 0.03$, and $A = 0.1$. $\theta_L \mid_{\mu=\mu^*}$, θ_M, and θ_T denote, respectively, the growth rates under lump-sum-tax financing and the Friedman Rule, money financing, and income tax financing.

PROPOSITION 2: *If* $A \leq 1$ *then* $\pi_M > \pi_T$.

PROOF: Notice that $d\mu_M/d\phi < 0$, $d\theta_M/d\phi < 0$, $d\theta_T/d\phi < 0$, and hence

$$(\pi_M)_{\min} = \mu_M|_{\phi=1} - \theta_M|_{\phi=0} = \frac{\gamma}{1-\gamma} - \frac{A-\rho}{\sigma},$$

$$(\pi_T)_{\max} = -\theta_T|_{\phi=0} = -\frac{(1-\gamma)A-\rho}{\sigma}.$$

Thus,

$$\pi_M - \pi_T \geq (\pi_M)_{\min} - (\pi_T)_{\max} = \gamma\left\{\frac{1}{1-\gamma} - \frac{A}{\sigma}\right\}.$$

Given $\gamma > 0$, the first term inside the brackets in the last expression is greater than one (if $\gamma = 0$, then $\pi_M = \pi_T$). If the second term is less than one, then the result follows. A sufficient condition for that is $A \leq 1$.■

Figure 5 depicts $(\pi_M)_{\min}$ and $(\pi_T)_{\max}$.

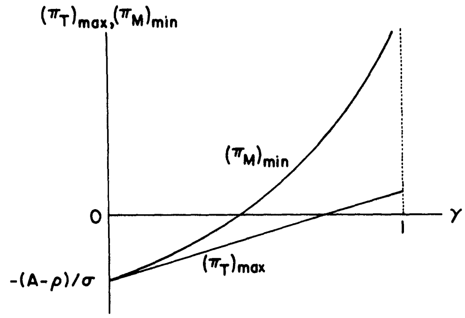

FIG. 5. Comparison between the Inflation Rates

4. GROWTH-MAXIMIZING TAX STRUCTURE

Thus far we have examined the case where the government relies *either* on seigniorage *or* on income taxation as a source of revenue. In this section we attempt instead to answer the following question. Given that the government has access to both sources of finance, what is the mix of these policies that yields the highest growth rate? We find that this consists only of seigniorage and not of income taxation. More formally,

PROPOSITION 3: *The values of τ and μ that maximize the economy's (positive) growth rate, given by (13), subject to the government budget constraint, (3), and the non-negativity constraints, $\tau \geq 0$ and $\mu \geq 0$, are $\tau = 0$ and $\mu = g/m = \gamma/[(1 - \gamma) - (1 - \phi)(\theta_M/A)]$.*

PROOF: Solving (13) for the unique positive θ, we get

$$\theta = \Omega/2a \ , \tag{18}$$

where $\Omega \equiv -b + (b^2 - 4ac)^{1/2} > 0$, $a \equiv \phi\sigma(\sigma - 1) > 0$, $b \equiv \sigma[1 + \phi(\rho + \mu)] + \rho\phi(\sigma - 1) > 0$ and $c \equiv \rho[1 + \phi(\rho + \mu)] - (1 - \tau)A < 0$. Furthermore, using (1) and (2), rewrite the government budget constraint as

$$\gamma = \mu[(1 - \gamma) - (1 - \phi)(\theta/A)] + \tau \ . \tag{19}$$

Differentiating (18) and using (13), it is straightforward to show that the slope of the level curves of the objective function is: $d\tau/d\mu|_{d\theta=0} = -[(\partial\Omega/\partial\mu)/(\partial\Omega/\partial\tau)] = -(\phi/A)(\rho + \sigma\theta) = -(\phi/A)[(1 - \tau)A/(1 + \phi R)] < 0$. Moreover, simple differentiation yields $d^2\tau/d\mu^2|_{d\theta=0} = 0$; thus, the level curves are downward-sloping straight lines (see Figure 6, where the arrow shows the direction in which θ increases).

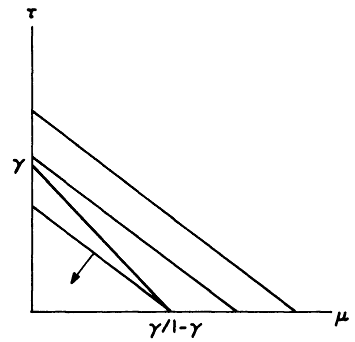

FIG. 6. Growth-Maximizing Tax Structure

It suffices then to show that the budget constraint is steeper than the level curves everywhere in the positive quadrant. That is,

$$\left| \left| \frac{d\tau}{d\mu} \right|_{d\theta=0} \right| < \left| \left| \frac{d\tau}{d\mu} \right|_{bc} \right| , \tag{20}$$

the absolute value of the slope of the level curves is lower than the absolute value of the slope of the budget constraint.

Consider first the case where $\phi = 1$. The budget constraint, (19), becomes a straight line $\gamma = \mu(1 - \gamma) + \tau$, and the absolute value of its slope is given by $|d\tau/d\mu|_{bc}| = 1 - \gamma = (1 - \tau)/(1 + \mu)$. Using the fact that $R > \mu$, one can easily show that condition (20) holds; thus, the optimal mix is the corner solution $\tau = 0$ and $\mu = g/m = \gamma/(1 - \gamma)$ (pure seigniorage). This case is depicted in Figure 6.

In the case where $0 \leq \phi < 1$, the slope of the constraint (19) is given by

$$d\tau/d\mu|_{bc} = -[(\partial\Omega/\partial\mu) - \beta\Delta]/[(\partial\Omega/\partial\tau) - \Delta] < 0 ,$$

where $\Delta \equiv [\mu(1 - \phi)/2aA]^{-1} > 0$ and $0 < \beta \equiv [(1 - \gamma)A - (1 - \phi)\theta]/A < 1$. Condition (20) is then equivalent to $Z \equiv (1 - \gamma)A - \phi\rho - [(\sigma - 1)\phi + 1]\theta > 0$. To prove $Z > 0$, rewrite (19) as $\theta = [A/(1 - \phi)]\{1 - \gamma - [(\gamma - \tau)/\mu]\}$ and notice that, for any $\mu \geq 0, \tau \geq 0$, $d\theta/d\phi > 0$. Hence, $dZ/d\phi = -[\rho + (\sigma - 1)\theta] - [\phi(\sigma - 1) + 1](d\theta/d\phi) < 0$. Thus, it suffices to show that $Z|_{\phi=1} = (1 - \gamma)A - (\rho + \sigma\theta) > 0$, or, since, as shown above, $\theta_M|_{\phi=1} > \theta_T|_{\phi=1}$, that $(1 - \gamma)A - (\rho + \sigma\theta_M) > 0$, which follows from Lemma 2 presented in the Appendix.■

5. WELFARE ANALYSIS

In this penultimate section we attempt to assess the welfare implications of various financing policies. Following the existing literature (for example, Barro 1990 and Turnovsky 1992, 1993), our welfare criterion consists of the lifetime utility, U, of the representative agent. Substituting the CEIS form of the instantaneous utility function and integrating the right-hand side of (4), taking into account that there are no transitional dynamics, yields U as a function of the economic growth rate (θ) and the initial consumption level ($c(0)$): $U = \Sigma + (1 - \sigma)^{-1} [c(0)]^{1-\sigma}[\rho + (\sigma - 1)\theta]^{-1}$, where $\Sigma \equiv 1/[\rho(\sigma - 1)]$. Furthermore, the private budget constraint, (1), the government budget constraint, (3), the money market equilibrium condition, $\dot{m}(t) = [\mu - \pi(t)]m(t)$, and equation (12) imply $c(0) = [(1 - \gamma)A - \theta]k(0)$.

Differentiation with respect to θ yields

$$\frac{dU}{d\theta} = \frac{k(0)[c(0)]^{-\sigma}}{[\rho + (\sigma - 1)\theta]^2} [(1 - \gamma)A - \rho - \sigma\theta] , \tag{21}$$

which implies that the growth rate that maximizes welfare is $\theta' \equiv [(1-\gamma)A - \rho]/\sigma$. The reason for this is that whenever an individual's decision raises per capita output by one unit, the government is induced to raise its spending by $\gamma = g/y$. The best way then to internalize this negative externality is to impose an income tax at the rate $\tau = \gamma = g/y$. Of course, if $\gamma = 0$, then $\theta' = \theta^*$, the first-best growth rate.

Moreover, notice that $\theta' = \theta_T|_{\phi=0}$. If, on the other hand, $\phi > 0$, then, as (13) suggests, the optimal growth rate, θ', can be achieved by a proportional tax at the rate γ and the Friedman rule, that is, $\theta' = \theta_T|_{\mu=\mu^*}$. Furthermore, Lemma 2, presented in the Appendix, shows that in the case where ϕ takes a particular value, ϕ', the optimal growth rate, θ', can also be achieved by the use of seigniorage. An intuitive explanation of this finding is as follows. In the case of income taxation, by applying the Friedman rule, the government neutralizes the negative effect on growth due to the CIA constraint; thus the only (negative) effect left is the one due to the presence of an income tax, which is, however, independent of ϕ. In the case of money financing, as ϕ increases and hence as the CIA constraint becomes more restrictive on investment, the negative effect of seigniorage on growth increases (the growth rate, θ_M, decreases). Therefore there must exist some fraction (ϕ') that equates the growth rates resulting from the two methods, that is, makes the two methods to have equally negative effects on growth. Under the parameter values specified in Table 1, $\phi' = 0.895$.

Next, we compare the welfare levels obtained under tax finance and seigniorage and the non-negativity constraints $\mu \geq 0$ and $\tau \geq 0$. As shown in Proposition 1, money financing of a given share of government spending results in a higher growth rate than does income tax financing. Nevertheless, at any given growth rate, the former method also results in a lower initial level of consumption than the latter. We have already shown that if $\phi = 0$ (ϕ') then tax-financing (seigniorage) is preferred. More generally, if we let U_M (U_T) denote the welfare level achieved under seigniorage (income taxation), then one can show

PROPOSITION 4: *There exists $\phi^* \in (0, \phi')$ such that $U_T \gtreqless U_M$ if $\phi \lesseqgtr \phi^*$. Furthermore ϕ^* is unique.*

PROOF: If $\phi = 0$, then $\theta' = \theta_T$ and hence $U_T > U_M$ (recall that θ' maximizes welfare). If, on the other hand, $\phi = \phi'$ then $\theta' = \theta_M$ and hence $U_T < U_M$. Furthermore, from (14) and (16), it follows that $d\theta_i/d\phi < 0$, $i = M, T$. Also, if $\phi \in (0, \phi')$ then $\theta_T < \theta' < \theta_M$ (see Lemma 2) so that equation (21) implies that $dU_T/d\theta > 0$ and $dU_M/d\theta < 0$. Hence, for $\phi \in (0, \phi')$, $dU_T/d\phi < 0$ and $dU_M/d\phi > 0$. We can then conclude that there exists a value of ϕ in the interval $(0, \phi')$, ϕ^*, such that $U_T = U_M$. Finally, if $\phi > \phi'$ then $\theta' > \theta_M > \theta_T$. Equation (21) then implies that $dU_i/d\theta > 0$ and thus $U_M|_{\phi=1} > U_T|_{\phi=1}$ and $dU_i/d\phi < 0$, $i = M, T$. From these relationships we can conclude that $U_M > U_T \; \forall \; \phi \in (\phi', 1]$. Hence ϕ^* is unique.■ A graphical illustration of this proposition is given in Figure 7. (Under the parameter values specified in Table 1, $\phi^* = 0.781$.)

Finally, rather than considering either tax finance or seigniorage, we examine the mix of the two policies that yields the highest welfare level. We find,

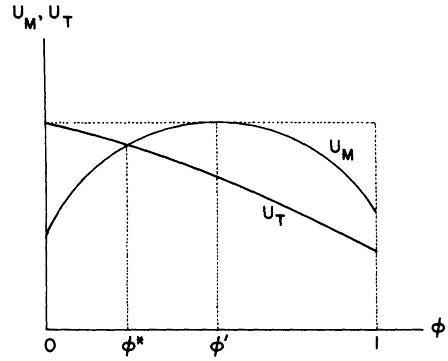

FIG. 7. Comparison between the Welfare Levels

PROPOSITION 5: *The values of* τ *and* μ *that maximize the representative agent's welfare, subject to the government budget constraint and the non-negativity constraints, are*

$$(i)\quad \textit{If } \phi \in [0, \phi'), \textit{ then } \mu = \frac{\phi}{(1-\phi)}(1-\gamma)A$$

$$\textit{and} \qquad \tau = \gamma - \frac{(1-\gamma)\phi}{\sigma}\left[\frac{\sigma - 1 + \phi}{1-\phi}(1-\gamma)A + \rho\right].$$

$$(ii)\quad \textit{If } \phi \in [\phi', 1], \textit{ then } \mu = g/m = \gamma/[(1-\gamma) - (1-\phi)(\theta_M/A)]$$

$$\textit{and} \qquad \tau = 0\,.$$

PROOF: (*i*) As shown above $\theta' \equiv [(1-\gamma)A - \rho]/\sigma$ maximizes welfare. If $\phi \in [0, \phi')$ then θ' can be achieved using a mix of the two financing methods. To find the exact values of μ and τ that can achieve θ', we substitute the expression for θ' in (13) and (19).

(*ii*) If $\phi \in [\phi', 1]$ then $\theta_T < \theta_M < \theta'$ and, as (21) implies, $dU_i/d\theta_i > 0$, $i = M, T$. Hence, maximizing the welfare level is equivalent to maximizing the growth rate. It follows then from Proposition 3 that the optimal mix involves only seigniorage.■

6. CONCLUDING REMARKS

We have analyzed the relative merits of seigniorage and income tax financing of government expenditure. From a growth perspective, our analysis suggests that seigniorage is always preferred. Income tax financing results, however, in a lower inflation rate. Finally, from a welfare perspective, the optimal policy depends crucially on the fraction of investment purchases that are subject to liquidity constraints.

Our analysis is subject to several qualifications which call for further research. For the sake of brevity, we outline just a few of them. As a country's inflation rate increases, there is typically a tendency to substitute foreign for local currency. This phenomenon, also known as dollarization, has been observed in several Central and South American countries, for example, Mexico, Argentina, and Panama. Clearly, in that case seigniorage as a method of expenditure financing is transferred from the domestic to the foreign country [see Fischer (1982) for a discussion]. An open-economy model which allows for currency substitution is therefore necessary for the study of this issue.

A relatively high inflation rate which results under alternative financing methods may affect different (heterogeneous) income groups in an asymmetric way. Nevertheless, our representative-agent framework abstracts from issues of income distribution.

Finally, within the "Ak" framework, used in this paper, there are no transitional dynamics and the economy jumps immediately to the balanced growth path. It would be interesting to examine the issues considered here in a more general model that exhibits transitional dynamics. While this would be unlikely to alter our conclusions regarding the balanced-growth-path comparison, it may affect the overall relative merits due to the presence of a transition path.

APPENDIX

CLAIM: *If* $\mu < \mu^* = -[\rho + (\sigma - 1)\theta]$ *then no equilibrium path exists.*

PROOF: Recall that along any equilibrium path $\pi = \mu - \theta$. Using this together with (10) and (11), one can write $\lambda_2 = \lambda_1[\rho + (\sigma - 1)\theta + \mu]$. The boundness of utility implies that $\rho > (1 - \sigma)\theta$. Thus, given that $\lambda_1 > 0$ and $d\theta/d\mu < 0$, if $\mu < \mu^*$, it follows that $\lambda_2 < 0$ which violates the Kuhn-Tucker condition that $\lambda_2(t) \geq 0\ \forall\ t$ [see (9)]. To see that $\lambda_1 > 0$, substitute (6), (10), (11), and $\pi = \mu - \theta$ in (5) to obtain $c^{-\sigma} = \lambda_1[1 + \rho + (\sigma - 1)\theta + \mu]$. Given then that (nominal) money balances always take a non-negative value and hence $\mu > -1$, it follows that $\lambda_1 > 0$.■

LEMMA 1: *Let* $F, G\colon \mathbb{R} \rightarrow \mathbb{R}$ *be differentiable on* $I \subseteq \mathbb{R}$ *with* $F'(\theta) < 0$, $F(\infty) = -\infty$, $G'(\theta) > 0$, *and* $F(0) > G(0)$. *Let also* $\tilde{\theta}$, θ_T, *and* θ_M *be such that* $G(\tilde{\theta}) = \eta$, $F(\theta_T) = \eta$, *where* $\eta \in (0,\infty)$, and $F(\theta_M) = G(\theta_M)$. *Furthermore, assume that* $\tilde{\theta} > \theta_T$. Then $\tilde{\theta} > \theta_M > \theta_T$.

PROOF: First notice that the existence of $\tilde{\theta}$, θ_T, and θ_M follows immediately from the properties of $F(\theta)$ and $G(\theta)$. Consider next the function $H(\theta)\colon [\theta_T, \tilde{\theta}] \subseteq \mathbb{R} \rightarrow J \subseteq \mathbb{R}$ defined as follows: $H(\theta) \equiv [G(\theta) - \eta] + [\eta - F(\theta)]$. Clearly, $H(\theta)$ is continuous on $[\theta_T, \tilde{\theta}]$, a connected set. Notice that $H(\theta_T) = G(\theta_T) - \eta \leq G(\tilde{\theta}) - \eta = 0$ since $G'(\theta) > 0$. Similarly, $H(\tilde{\theta}) = \eta - F(\tilde{\theta}) \geq \eta - F(\theta_T) = 0$, since $F'(\theta) < 0$. Thus, $H(\theta_T) \leq 0 \leq H(\tilde{\theta})$. But then, by Bolzano's Intermediate Value Theorem, $\exists\ \theta \equiv \theta_M \in [\theta_T, \tilde{\theta}]$ such that $H(\theta_M) = 0$ or $F(\theta_M) = G(\theta_M)$.■

LEMMA 2: *There exists* $\phi' \in (0, 1)$ *such that* $\theta_M \gtreqless \theta_T|_{\mu=\mu^*}$ *if* $\phi \lesseqgtr \phi'$.

PROOF: Recall that $\theta_T|_{\mu=\mu^*} = \theta' \equiv [(1-\gamma)A - \rho]/\sigma$ (see section 5) and that, from equation (16), $\theta_M|_{\phi=0} = (A-\rho)/\sigma$. Therefore, $\theta_M|_{\phi=0} > \theta_T|_{\mu=\mu^*}$, for $\gamma > 0$. Next consider the case where $\phi = 1$. Let $\bar{F}(\theta)$ denote the left-hand side and $\bar{G}(\theta)$ denote the right-hand side of (16), when $\phi = 1$. $\theta_M|_{\phi=1}$ is defined as follows: $\bar{F}(\theta_M|_{\phi=1}) = \bar{G}(\theta_M|_{\phi=1})$. Furthermore notice that $\bar{G}(\theta_T|_{\mu=\mu^*}) = A\gamma \in (0,\infty)$ and that (if $\phi = 1$) $\theta_T|_{\mu=\mu^*} = \tilde{\theta}$. Also, $\theta_T|_{\mu=\mu^*} > \theta_T$ for $\phi > 0$ and $\bar{F}(\theta_T) = A\gamma$ [from (14)]. By applying Lemma 1 then $\theta_T|_{\mu=\mu^*} > \theta_M|_{\phi=1}$. Hence, we have shown that $\theta_M|_{\phi=0} > \theta_T|_{\mu=\mu^*}$ and $\theta_M|_{\phi=1} < \theta_T|_{\mu=\mu^*}$. Given that $d\theta_M/d\phi < 0$ and $d\theta_T|_{\mu=\mu^*}/d\phi = 0$, the result follows.■

LITERATURE CITED

Abel, Andrew B. "Dynamic Behavior of Capital Accumulation in a Cash-in-Advance Model." *Journal of Monetary Economics* 16 (July 1985), 55–72.

Alogoskoufis, George, and Frederick van der Ploeg. "On Budgetary Policies, Growth and External Deficits in an Interdependent World." *Journal of the Japanese and International Economies* 5 (December 1991), 305–24.

Barro, Robert J. "Government Spending in a Simple Model of Endogenous Growth." *Journal of Political Economy* 98 (October 1990), S103–25.

Cagan, Phillip. "The Monetary Dynamics of Hyperinflation." In *Studies in the Quantity Theory of Money*, edited by Milton Friedman, pp. 25–117. Chicago: University of Chicago Press, 1956.

Cooley, Thomas F., and Gary D. Hansen. "Tax Distortions in a Neoclassical Monetary Economy." *Journal of Economic Theory* 58 (December 1992), 290–316.

Dornbusch, Rudiger, and Jacob A. Frenkel. "Inflation and Growth: Alternative Approaches." *Journal of Money, Credit, and Banking* 5 (May 1973), 141–56.

Dornbusch, Rudiger, and Stanley Fischer. "Moderate Inflation." National Bureau of Economic Research Working paper no. 3896, 1991.

Fischer, Stanley. "Seigniorage and the Case for a National Money." *Journal of Political Economy* 90 (April 1982), 295–313.

Friedman, Milton, *The Optimal Quantity of Money and Other Essays*. Chicago: Aldine, 1969.

Haliassos, Michael, and James Tobin. "The Macroeconomics of Government Finance." In *Handbook of Monetary Economics*, edited by Benjamin M. Friedman and Frank Hahn, pp. 889–959. New York: Elsevier Science Publishers, 1990.

Hall, Robert E. "Intertemporal Substitution in Consumption." *Journal of Political Economy* 96 (April 1988), 339–57.

Helpman, Elhanan, and Efraim Sadka. "Optimal Financing of the Government's Budget: Taxes, Bonds, or Money?" *American Economic Review* 69 (May 1979), 152–60.

Jones, Larry E., and Rodolfo Manuelli. "Growth and the Effects of Inflation." National Bureau of Economic Research Working paper no. 4523, 1993.

Kimbrough, Kent P. "The Optimal Quantity of Money Rule in the Theory of Public Finance." *Journal of Monetary Economics* 18 (November 1986), 277–84.

King, Robert G., Charles I. Plosser, and Sergio T. Rebelo. "Production, Growth and Business Cycles: I. The Basic Neoclassical Model." *Journal of Monetary Economics* 21 (March/May 1988), 195–232.

King, Robert G., and Sergio T. Rebelo. "Public Policy and Economic Growth." *Journal of Political Economy* 98 (October 1990), S126–51.

Koenig, Evan F. "The Short-Run 'Tobin Effect' in a Monetary Optimizing Model." *Economic Inquiry* 25 (January 1987), 43–54.

———. "Investment and the Nominal Interest Rate: The Variable Velocity Case." *Economic Inquiry* 27 (April 1989), 325–44.

Lucas, Robert E., Jr. "Equilibrium in a Pure Currency Economy." *Economic Inquiry* 18 (April 1980), 203–20.

———. "On the Mechanics of Economic Development." *Journal of Monetary Economics* 22 (July 1988), 3–42.

Mino, Kazuo. "Money and Endogenous Growth in a Cash-in-Advance Economy." Manuscript, Tohoku University, 1991.

Palivos, Theodore, Ping Wang, and Jianbo Zhang. "Velocity of Money in a Modified Cash-in-Advance Economy: Theory and Evidence." *Journal of Macroeconomics* 15 (Spring 1993), 225–48.

———. "On the Existence of Balanced Growth Equilibrium." Manuscript, 1994.

Phelps, Edmund S. "Inflation in the Theory of Public Finance." *Swedish Journal of Economics* 75 (January/March 1973), 67–82.

Ploeg, Frederick van der, and George Alogoskoufis. "Money and Endogenous Growth." *Journal of Money, Credit, and Banking* 26 (November 1994), 771–91.

Rebelo, Sergio T. "Long-Run Policy Analysis and Long-Run Growth." *Journal of Political Economy* 99 (June 1991), 500–21.

Romer, Paul M. "Increasing Returns and Long-Run Growth." *Journal of Political Economy* 94 (October 1986), 1002–37.

———. "Capital Accumulation in the Theory of Long-Run Growth." In *Modern Business Cycle Theory*, edited by Robert J. Barro, pp. 51–127. Cambridge, Harvard University Press, 1989.

Sala-i-Martin, Xavier. "Lecture Notes on Economic Growth: Five Prototype Models of Endogenous Growth II." National Bureau of Economic Research Working paper no. 3655, 1990.

Stockman, Alan C. "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy." *Journal of Monetary Economics* 8 (November 1981), 387–93.

Turnovsky, Stephen J. "Alternative Forms of Government Expenditure Financing: A Comparative Welfare Analysis." *Economica* 59 (May 1992), 235–52.

———. "Macroeconomic Policies, Growth, and Welfare in a Stochastic Economy." *International Economic Review* 34 (November 1993), 953–81.

Turnovsky, Stephen J., and William A. Brock. "Time Consistency and Optimal Government Policies in Perfect Foresight Equilibrium." *Journal of Public Economics* 13 (April 1980), 183–212.

Wang, Ping, and Chong K. Yip. "Alternative Approaches to Money and Growth." *Journal of Money, Credit, and Banking* 24 (November 1992), 553–62.

———. "Real Effects of Money and Welfare Costs of Inflation in an Endogenously Growing Economy with Transactions Costs." Manuscript, Penn State University, 1993.

Weber, Warren E. "Interest Rates, Inflation, and Consumer Expenditures." *American Economic Review* 65 (December 1975), 843–58.

Woodford, Michael. "The Optimum Quantity of Money." In *Handbook of Monetary Economics*, edited by Benjamin M. Friedman and Frank Hahn, pp. 1067–152. New York: Elsevier Science Publishers, 1990.