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*Velocity of Money in a Modified Cash-in-Advance Economy: Theory and Evidence**

We develop a modified cash-in-advance model in which money is required prior to all the purchases of the consumption good and of a fraction of the capital good. This framework enables us to examine the main determinants of money velocity in a dynamic general equilibrium setting. We find that, contrary to standard beliefs, high-money-growth equilibria are associated with lower welfare and lower velocity than low-money-growth equilibria. This result is supported by empirical evidence, both over time and across countries.

1. Introduction

Due to its important empirical implications, the study of money velocity has interested economists for years. In their classic work Friedman and Schwartz (1963) attribute the movement in money velocity to factors affecting money demand as well as to the growing financial sophistication of the economy. Recently, Bordo and Jonung (1981, 1987, 1989) have extensively studied the long-run effects of institutional factors, such as financial development, on the velocity of money. Also, Rasche (1987) reconsiders the behavior of money velocity and examines whether shifts in wealth, interest rates, tax rates, trade deficits, and monetary policy variables lead to significant changes in it. However, the theoretical hypotheses identifying the movements in money velocity have not yet been completely

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developed in the context of a general equilibrium model. In particular, most of the empirical work on money velocity relies on money demand theory in which interactions between the real and the monetary sector are omitted.

In this paper, we establish a dynamic general equilibrium model in which money is introduced through a modified cash-in-advance constraint. An unattractive characteristic of the traditional cash-in-advance models with homogeneous goods, in a deterministic setting, is that the velocity of money is independent of changes in the money growth rate. More specifically, if capital is abstracted from the cash-in-advance model (compare Lucas 1980), one obtains a unitary velocity. By allowing for capital accumulation but assuming that money is required only for the purchases of the consumption good (compare Stockman 1981), money becomes superneutral, and hence any money growth policy will not affect money velocity. In the latter case, if money is required for the purchases of the capital good as well (compare Stockman 1981), then velocity becomes again identically one. Even within a stochastic framework (for example, see Svensson 1985), money growth may change the velocity of money, only when the cash-in-advance constraint is not binding (since in this case, long-run movements in the growth rate of money may affect real money balances and output by a different magnitude).

To allow for possible linkage between money growth and velocity in the long run, we postulate that only a fraction, θ , of the capital good is subject to the cash-in-advance constraint. This modification enables us to study the determinants of money velocity, including monetary policy, various financial institutional changes, and an output technological factor. We find that, contrary to findings in previous work, high-money-growth equilibria are associated with lower welfare and lower velocity than low-money-growth equilibria when this fraction (θ) is given a priori. This occurs because a higher money growth rate creates an adverse effect on capital accumulation (that is, the reverse Tobin effect) and consumption. Under decreasing returns to scale, such an effect is found to suppress money velocity. One may also allow the fraction θ to depend on the endogenously determined inflation rate and on an exogenous factor capturing institutional changes in the financial system. In this case, if a higher money growth rate and the associated higher inflation make credit trading (of the capital good) less feasible and hence increase the fraction of the capital good subject to the cash-in-advance constraint, then there will be an additional downward pressure on the velocity of money.

Moreover, we study the long-run effects of technological improvement and financial development on macroeconomic aggregates and on the velocity of money. We show that any policy enhancing credit trading will result in higher money velocity. On the other hand, a technological improvement in production will increase both real income and real money balances, thus creating an ambiguous effect on the velocity of money. We also provide empirical evidence to lend support to our main theoretical results and find that there is a consistently negative relationship between the rate of money growth and money velocity, both over time and across countries. Finally, it should be noted that one cannot draw any conclusions from our paper to address this issue in the case of hyperinflation in which the transactions frequency is expected to change significantly over time (for details, see Section 6 below).

The remainder of the paper is organized as follows. Section 2 develops a modified cash-in-advance model, Section 3 characterizes the steady-state solution, and Section 4 demonstrates the determinants of money velocity. Empirical evidence is then provided in Section 5 and conclusions are drawn in Section 6.

2. The Model

In this section, we consider a dynamic, general equilibrium model in which money is introduced through a cash-in-advance constraint.

Preferences and Technology

A representative agent with perfect foresight seeks to maximize his/her lifetime utility described by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad (1)$$

where c is per capita consumption and $\beta \in (0, 1)$ is the constant discount rate. The felicity function, $u(\cdot)$, is assumed to be increasing, twice continuously differentiable, strictly concave, and to satisfy the Inada conditions, that is, $u_c(0) = \infty$ and $u_c(\infty) = 0$. In each period t , the agent produces a certain amount of output, y_t , by using a common technology described by $A_t f(k_t)$, where k_t denotes per capita capital stock and A_t is a multiplicative technological parameter. The capital stock is assumed to be fully depreciated at the

end of each period.¹ Further, the production function is assumed to be increasing, twice continuously differentiable, strictly concave, and to satisfy $f(0) = 0$ and the Inada conditions. In addition, at the beginning of each period the individual receives a lump sum cash transfer of amount τ_t (expressed in real terms) from the government. Given any amount of income, the individual can consume it during this period, save it in the form of money, or invest it to produce in the next period.

Constraints

Let P_t be the price level in period t and M_t be the nominal money holdings in the beginning of period t . Next, by defining real money holdings in the beginning of period t as $m_t = M_t/P_{t-1}$, we can write the agent's budget constraint in real terms as:²

$$c_t + m_{t+1} + k_{t+1} = A_t f(k_t) + \frac{m_t}{1 + \pi_t} + \tau_t, \quad (2)$$

where π_t denotes the inflation rate defined as $\pi_t = (P_t - P_{t-1})/P_{t-1}$.

The individual is also subject to a cash-in-advance or liquidity constraint. That is, all the purchases of the consumption good, as well as a fraction θ of the investment good, can be made only by using money.³ The fraction $(1 - \theta)$ of the investment good, on the other hand, is purchased on credit. The same distinction between "cash" and "credit" goods can also be found in Lucas and Stokey (1987), although in the absence of capital, their credit good enters the agent's utility function directly. It has been claimed, however, that this distinction is unlikely to be independent of the inflation rate (Blanchard and Fischer 1989). To circumvent this problem, one can allow θ to depend on the inflation rate. In general, θ_π has an ambiguous sign, since any change in the inflation rate will shift both the supply of and the demand for loanable funds. Nevertheless, if

¹For an empirical investigation of the behavior of M1-velocity using the VAR approach see McMillin (1991).

²This assumption is made only for convenience; all the main results, derived below, remain qualitatively the same, as long as the rate of return on capital is positive; that is, $Af_k > \delta$, where δ denotes the depreciation rate.

³The budget constraint in nominal terms is $P_t c_t + P_t k_{t+1} + M_{t+1} = P_t A_t f(k_t) + M_t + P_t \tau_t$. By dividing both sides of this equation by P_t , we obtain Equation (2). It should also be noted that m_{t+1} and k_{t+1} measure, respectively, money and capital demands in period t while $M_t/P_t = m_t/(1 + \pi_t)$ denotes initial real money holdings evaluated at the end of period t .

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the effect on the supply of funds is dominant, then one can expect θ_π to be positive.⁴

The fraction θ is also allowed to depend on an exogenous credit enhancement measure, ϕ , where $\theta_\phi < 0$. Any credit enhancement policy, captured by an increase in ϕ , will result in a decrease in θ . Examples include the use of L/C (letter of credit) and credit guarantee, in trade and financial markets, respectively.⁵ Thus, the following liquidity constraint must be satisfied in each period:

$$c_t + \theta(\pi_t, \phi_t)k_{t+1} \leq \frac{m_t}{1 + \pi_t} + \tau_t. \quad (3)$$

In summary, the representative agent seeks to maximize (1) by choosing his/her consumption path, subject to the resource and the cash-in-advance constraints, (2) and (3), respectively, given the path of prices and government transfers.

Individual's Optimization

We next turn to the individual's maximization problem to obtain the efficiency condition for the consumption-investment decision. Then, by combining this with the government budget constraint and the money supply process, to be specified below, we obtain the dynamic system that completely describes this economy.⁶ Ultimately, we will be interested in the steady-state solution of the system. Thus, from this point, we take the cash-in-advance constraint to be binding.⁷

⁴The cases $\theta = 0$ and $\theta = 1$ are associated with Lucas (1980) and Stockman (1981), respectively. Nevertheless, Stockman characterized both cases: he demonstrated that money is superneutral only in the former case. One could also allow for a fraction of the consumption good to be subject to the liquidity constraint. This however would complicate the model without changing the results.

⁵As mentioned below (see the section titled "Empirical Results from Individual Countries"), in most of the countries we examine the correlation between money growth rate and real money balances is found to be non-negative. In terms of our model, this finding is consistent with $\theta_\pi > 0$.

⁶Other financial developments that encourage the use of cash, such as the establishment of NOW and SUPERNOW accounts, can be viewed as a reduction in ϕ (which will lead to a higher θ and hence higher level of money holdings). For a discussion on the link between financial development and velocity see Friedman and Schwartz (1963), Bordo and Jonung (1987).

⁷The absence of any distortionary taxation makes the solution to the social planner's problem, on which we concentrate, identical to the one derived in a competitive economy. Thus, any internal debt generated by credit trading can be ignored by Walras' law (see Lucas and Prescott 1971).

Let $W_t(m_t, k_t)$ be the value function in period t , given the state variables k_t and m_t . Then, by Bellman's principle of optimality, we have

$$W(m_t, k_t) = \max_{c_t} \{u(c_t) + \beta W(m_{t+1}, k_{t+1})\}, \quad (4)$$

subject to (2) and (3).

The first-order condition for an optimal consumption stream c_t , $t = 0, 1, \dots$, is found to be

$$u_c(c_t) = \beta \left\{ \left(1 - \frac{1}{\theta(\pi_t, \phi_t)} \right) W_m(m_{t+1}, k_{t+1}) + \frac{1}{\theta(\pi_t, \phi_t)} W_k(m_{t+1}, k_{t+1}) \right\}. \quad (5)$$

Also, the Beveniste-Scheinkman (1979) equations for the evolution of the state variables are

$$W_k(m_t, k_t) = \beta W_m(m_{t+1}, k_{t+1}) A_t f_k(k_t), \quad (6)$$

and

$$W_m(m_t, k_t) = \frac{\beta}{1 + \pi_t} \left\{ \left(1 - \frac{1}{\theta(\pi_t, \phi_t)} \right) W_m(m_{t+1}, k_{t+1}) + \frac{1}{\theta(\pi_t, \phi_t)} W_k(m_{t+1}, k_{t+1}) \right\}. \quad (7)$$

By using (5), Equations (6) and (7) can alternatively be written as

$$W_k(m_t, k_t) = \frac{1}{1 + \pi_t} \beta A_t f_k(k_t) u_c(c_{t+1}), \quad (8)$$

and

$$W_m(m_t, k_t) = \frac{1}{1 + \pi_t} u_c(c_t). \quad (9)$$

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Next, updating (8) and (9), and substituting them into (5) one obtains the following optimization condition:

$$\begin{aligned} \theta(\pi_t, \phi_t)u_c(c_t) + [1 - \theta(\pi_t, \phi_t)]\beta \frac{1}{1 + \pi_{t+1}} u_c(c_{t+1}) \\ = \beta^2 \frac{1}{1 + \pi_{t+2}} A_{t+1} f_k(k_{t+1}) u_c(c_{t+2}) . \end{aligned} \quad (10)$$

Suppose the individual decreases investment in period t by one unit. This frees up θ units of real balances which can be used for consumption in period t (the first term on the LHS) while the remainder $(1 - \theta)$ can be held in the form of real balances to finance consumption in period $t + 1$ (the second term on the LHS). The RHS expresses the loss in utility due to forgone consumption in period $t + 2$ that the proceeds of investment would have purchased. According to Equation (10), at the optimum and after accounting for inflation and discounting for the future, the loss in utility must be equal to the gain.

Government and Market Equilibrium

To close the model, we next consider a benevolent government facing the following budget constraint:⁸

$$\tau_t = m_{t+1} - \frac{m_t}{1 + \pi_t} . \quad (11)$$

Let μ be the constant money growth rate. For simplicity, the money supply process is assumed to be $M_{t+1} = (1 + \mu) M_t$, which together with the money market equilibrium condition imply

$$m_{t+1} = [(1 + \mu)/(1 + \pi_t)] m_t . \quad (12)$$

Substituting Equations (11) and (12) into the private budget constraint (2), we can derive the goods market equilibrium condition:

$$c_t + k_{t+1} = A_t f(k_t) . \quad (13)$$

⁸By use of the Kuhn-Tucker theorem, one can show that in steady state the cash-in-advance constraint has to be binding. The proof is very similar to Stockman (1981) and thus omitted.

Equations (10), (11), (12), (13) and (3), with equality, constitute a dynamic system that completely characterizes the equilibrium path of this economy.

3. Characterization of the Steady-State Solution

In this section, we look at the system in its steady-state position, in which, given a constant money growth rate, all the endogenous variables are constant (that is, $c_t = c$, $k_t = k$, and $m_t = m$, for all t).

Steady-State Solution

In the steady state, the dynamic system simplifies to

$$c + k = Af(k), \quad (14)$$

$$\mu = \pi, \quad (15)$$

$$c + \theta(\pi, \phi)k = \frac{m}{1 + \pi} + s, \quad (16)$$

$$\tau = \frac{\pi}{1 + \pi} m, \quad (17)$$

$$Af_k(k) = \frac{1 + \mu - \beta}{\beta^2} \theta(\pi, \phi) + \frac{1}{\beta}. \quad (18)$$

Equation (15) is a well-known result; namely, in steady-state equilibrium and in the absence of population growth, the inflation rate equals the money growth rate. Also, Equation (18) is analogous to the modified golden rule condition. Indeed, for $\theta(\pi, \phi) = 0$ it becomes $Af_k(k) = 1/\beta$, which is the modified golden rule condition in an economy with fully depreciated capital and constant population size.

Comparative Statics

By substituting (14) and (17) into (16) we obtain the following equation:

$$Af(k) + [\theta(\pi, \phi) - 1]k = m. \quad (19)$$

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Further, we differentiate (18) and (19) to get

$$\begin{bmatrix} \beta^2 A f_{kk} & 0 \\ A f_k + \theta - 1 & -1 \end{bmatrix} \begin{bmatrix} dk \\ dm \end{bmatrix} = \begin{bmatrix} -\beta^2 f_k & (1 + \mu - \beta)\theta_\phi & \Lambda \\ -f & -\theta_\phi k & -\theta_\pi k \end{bmatrix} \begin{bmatrix} dA \\ d\phi \\ d\mu \end{bmatrix}, \quad (20)$$

where $\Lambda = (1 + \mu - \beta)\theta_\pi + \theta > 0$ if $\theta_\pi > -\theta/(1 + \mu - \beta)$, which is of course satisfied in the case where $\theta_\pi \geq 0$. Let D be the determinant of the premultiplied matrix of vector $[dk \ dm]'$ which can be shown to be positive. Note, also, that under Diamond efficiency, which implies $dc/dk > 0$, $A f_k > 1$. Then, straightforward comparative statics analysis yields

$$\begin{aligned} \frac{dk}{dA} &= \frac{1}{D} \beta^2 f_k > 0, \\ \frac{dk}{d\phi} &= \frac{-1}{D} (1 + \mu - \beta)\theta_\phi > 0, \\ \frac{dk}{d\mu} &= \frac{-1}{D} \Lambda < 0, \\ \frac{dm}{dA} &= \frac{\beta^2}{D} [f_k(A f_k + \theta - 1 - f_{kk}) - A f f_{kk}] > 0, \\ \frac{dm}{d\phi} &= \frac{-\theta\phi}{D} [(1 + \mu - \beta)(A f_k + \theta - 1) + \beta^2 f_{kk} k] \cong 0, \\ \frac{dm}{d\mu} &= \frac{-1}{D} [\Lambda(A f_k + \theta - 1) + \beta^2 A f_{kk} k \theta_\pi] \cong 0. \end{aligned} \quad (21)$$

These, together with (14), lead to

$$\frac{dc}{dA} > 0, \quad \frac{dc}{d\phi} > 0, \quad \frac{dc}{d\mu} < 0. \quad (22)$$

Intuitively, a technological improvement increases the marginal product of capital and thus increases the steady-state level of capital. Under Diamond efficiency, consumption will increase as well. Hence, to finance a higher level of consumption and investment, real money balances must also increase.

Any financial improvement which decreases θ , and is captured by an increase in ϕ , has a direct negative effect on real money holdings and results in a lower marginal cost of capital. This leads to higher levels of consumption and capital, which in turn require more money to facilitate transactions. Overall, the net effect on money holdings is ambiguous.

Finally, money is not superneutral. More specifically, higher rates of money growth are likely associated with lower steady-state levels of capital (that is, reverse Tobin effect) and consumption, thus decreasing social welfare.⁹ It is important to distinguish again between the two different effects that may lead to this result. A higher rate of money growth implies a higher rate of inflation. Even with an exogenous θ (that is, $\theta_\pi = 0$), an increase in inflation will raise the cost of money holdings and thus it will decrease the *net rate of return* on capital. This will cause in turn a decrease in investment and consumption. However, θ can be allowed to depend on the endogenously determined inflation rate. In the case of $\theta_\pi > 0$ an increase in inflation will also increase θ , which is equivalent to an increase in the shadow cost of capital. Thus, the level of investment and hence consumption will fall even more. This reverse Tobin effect can be obtained even if $\theta_\pi < 0$. A sufficient condition for this is: $\theta_\pi > -\theta/(1 + \mu - \beta)$. If, however, an increase in inflation causes a sufficient increase in the fraction of the investment good purchased on credit, then the shadow cost of capital will decrease. In this case the Tobin effect (see Tobin 1965) will be present contrary to the standard cash-in-advance models. Finally, as in the case of a change in ϕ , the effect of money growth expansion on the level of real money balances is ambiguous.¹⁰

4. Velocity of Money

This section examines the determinants of the income velocity of money and elaborates on related issues. Specifically, we study

⁹In nominal terms, this constraint is $P_t \tau_t = M_{t+1} - M_t$.

¹⁰These should be viewed as anticipated changes in the money growth rate, similar to Brock (1975).

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how money velocity is affected by changes in the exogenous variables: A , ϕ , and μ .

Determinants of Money Velocity

In terms of the theoretical model, the velocity of money is defined as

$$V \equiv \frac{c + k}{m} = \frac{c + k}{c + \theta k}. \quad (23)$$

Thus higher θ , *ceteris paribus*, decreases money velocity.

By totally differentiating (23) and using (21) one obtains the additional comparative statics results:¹¹

$$\frac{dV}{dA} \cong 0, \quad \frac{dV}{d\phi} > 0, \quad \frac{dV}{d\mu} \cong 0. \quad (24)$$

As mentioned in Section 3, an improvement in the technological parameter (A) will increase the steady-state levels of capital, consumption, and real money balances. Hence, the effect on velocity is indeterminate.

Any credit enhancement policy, captured by an increase in ϕ , will create a direct downward effect on θ and thus it will increase velocity through the reduction of money holdings. It will also increase consumption and capital accumulation, as indicated in (21) and (22), which will in turn lead to a further increase of money velocity, under the presumption of $\theta < 1$. Therefore, money velocity will unambiguously increase.

Finally, the velocity of money may depend negatively on the money growth rate and thus it may be inversely related to the inflation rate. Notice that in a traditional monetary model, an increase

¹¹We have performed a calibration exercise to study this issue using the functional forms $f(k) = k^a$ and $\theta(\pi, \phi) = \theta(\pi, 1) = [(\pi/1 + \pi)]^b$ and the following parameter values: $a = 0.28$ (the average labor income share in the U.S. computed from NIPA accounts), $\beta = 0.988$ and $r = 6.5\%$ (both taken from King et al. 1988), $b = 0.15$ (computed from Equation [18], using $\mu = 6.54\%$, which is the average value in the U.S. over the period 1963–1985), and $A^{1/(1-a)} = 1.65$ (calculated in such a way that the output level corresponding to $\mu = 0$ is normalized to one). We have found that, as μ rises, real balances initially increase due to the direct effect of θ , reach a peak at approximately $\mu = 11\%$, and eventually decrease because of the high opportunity cost of holding money.

in the rate of inflation increases the cost of money holdings, and thus money demand falls relative to output. Our cash-in-advance model takes this reasoning one step further, since it emphasizes also the transactions demand for money (people need cash to finance real purchases of consumption and capital good). To acquire a better understanding of the negative relationship between money growth rate and money velocity we manipulate (23) to obtain

$$V = \frac{1}{1 - (1 - \theta)k/y}, \quad (25)$$

where $y = c + k$. By differentiating (25) we have

$$\frac{dV}{d\mu} = \frac{(1 - \theta)[d(k/y)/d\mu] - \theta_\pi(k/y)}{[1 - (1 - \theta(\pi, \phi))(k/y)]^2} \leq 0, \quad (26)$$

where $d(k/y)/d\mu = [(f - kf_k)/Af^2] (dk/d\mu) < 0$, under the assumption of decreasing returns to scale in the per capita output production function, that is, $f - kf_k > 0$.

To be more specific, an increase in the money growth rate will in general have two effects on money velocity, one through the cost of money holdings and the other through θ . First, as we argued in Section 3, a higher money growth rate is associated with a higher rate of inflation which will increase the cost of money holdings and thus decrease investment. Under the assumption of decreasing returns to scale in the per capita output production function, a decrease in investment will result in a decline of the capital-output ratio and thus of money velocity (see Equation [25]). This effect will be present if $\theta_\pi = 0$, that is, θ is independent of the inflation rate. Note however that it will disappear if $\theta = 1$ (as in Stockman [1981]). Second, if we allow θ to depend on the inflation rate, and, in particular, if θ_π is positive, then there will be an additional downward pressure on velocity (recall from Section 3, that an increase in θ , resulting from an increase in the inflation rate, will have a negative effect on k and c and a positive one on real money holdings). Finally, notice that if θ_π is negative and sufficiently large in absolute value, then the opposite result may emerge.

Equation (25) can be rewritten as:

$$V = V(\theta, MPK), \quad V_1 \leq 0, \quad V_2 < 0, \quad (27)$$

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where *MPK*, the marginal product of capital, is negatively correlated with the capital-output ratio. Thus, all the effects mentioned above can be decomposed into effects through θ and through *MPK*. By using (21), one can obtain the following reduced form equation:

$$V = g(\mu, \phi, A), \quad (28)$$

where $g_1 \leq 0$, $g_2 > 0$ and $g_3 \leq 0$.

Further Discussion

Traditionally, based on Baumol (1952), and Tobin (1956), the velocity of money is expressed as a function of real income and nominal interest rate (*i*). That is,

$$V = h(y, i)$$

where

$$h_1 > 0, \quad h_2 > 0,$$

or

$$V = h(y, r + \pi), \quad (29)$$

where *r* is the real interest rate and is analogous to *MPK* in (27). Nevertheless, (29) is derived within a partial equilibrium framework in which both *y* and *r* are not endogenized. In fact, this is exactly the point that we try to emphasize in this section, that is, that *ceteris* will not remain *paribus* and that both *y* and *r* in Equation (29) will change as μ and thus π change.

5. Empirical Evidence

In this section, we examine more closely the inverse relationship between velocity and money growth rate and draw some evidence from the U.S. and several other countries as well.

The Data

We choose a group of twenty representative countries for our econometric analysis. The selection of countries is based upon the following criteria:

- (i) the availability of data during the sample period, 1963–1985;
- (ii) the quality of the data (see Summers and Heston 1988);
- (iii) co-movements between money growth and inflation rates, in order to satisfy the equality $\mu = \pi$ (implied by our theoretical model), a necessary condition for a country to be on a long-run equilibrium path.¹² The satisfaction of this criterion is based on the data provided in Barro (1990, Table 7.1);¹³ and
- (iv) the comprehensiveness of the data reflecting the characteristics of the underlying economy and, in particular, economic development (income per capita) and stabilization (inflation rate).

Table 1 reports the classification of the selected countries, according to their per capita GNP (low, lower-middle, upper-middle, and high) and their average inflation rates measured by the GDP

¹²A sufficient condition for these results is the envelope property; namely that $f(k) - kf_k > 0$, which is implied by the conditions $f_f > 0$, $f_{kk} < 0$, and $f(0) = 0$.

¹³In the case where there is technical progress at a rate η , this condition becomes $\mu = \pi + \eta$. Nevertheless, the rate of technical progress is in general small and thus can be ignored without introducing significant bias.

TABLE 1. *Classification of Selected Countries*

Inflation Rate	Per capita Income			
	Low	Lower-middle	Upper-middle	High
High	Zaire	Bolivia	Brazil	Israel
Medium	Madagascar	Philippines	Portugal	Ireland
	Ghana	Turkey	Mexico	Spain
Low	Haiti	Morocco	Venezuela	U.S.
	India	Thailand	South Africa	U.K.

NOTES: (a) Classification of per capita income (U.S.\$ in 1986)

- Low — below 400
- Lower-middle — 401 to 1500
- Upper-middle — 1501 to 4500
- High — above 4500

(b) Classification of inflation rate (%)

- Low — below 9.0
- Medium — 9.0 to 25.0
- High — above 25.0

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deflator (low, middle, and high).¹⁴ Geographically, this sample includes 6 countries from America (Bolivia, Brazil, Haiti, Mexico, Venezuela, and the U.S.), 5 from Africa (Zaire, Madagascar, Ghana, Morocco, and South Africa), 5 from Europe (Turkey, Portugal, Ireland, Spain, and the U.K.), and 4 from Asia (India, Philippines, Thailand, and Israel). Their values of per capita GNP range from 160 U.S.\$ in 1986 (Haiti) to 17,480 (U.S.), while their annual inflation rates fall between 5.38% (Thailand) and 55.92% (Bolivia).

The output measure is real GDP and the money supply measure is the monetary base. Money velocity is therefore defined as the ratio of nominal GDP to monetary base.¹⁵

Since the money growth rate in our theoretical framework is exogenous to the individual's optimization problem, we use the monetary base in lieu of M1. Nevertheless, by comparing different measures of money supply (currency, monetary base, and M1), we find that movements in the associated velocity measure are generally similar. For the twenty selected countries, the average money growth rates range between 6.47% (U.S.) and 57.83% (Bolivia), and the average money velocity measures fall in between 1.27 (Israel) and 18.17 (South Africa). Among all the countries, Brazil has the most volatile money velocity (the standard deviation of the mean is 1.30), while Portugal's money velocity is the smoothest (the standard deviation of the mean is 0.14)—see Table 2 for a summary of main indicators.

Empirical Results from Individual Countries

We next perform a time-series analysis for each individual country. In the absence of precise measures of financial and economic development, we incorporate real GDP (RGDP) into the regression of money velocity (VM), as well as the key variable—the rate of money growth (GRM).¹⁶ The regression is specified as

¹⁴Except for a few cases where we have no alternative, we restrict $z = (\mu - \pi)/[(\mu + \pi)/2]$ to be less than 0.3.

¹⁵The high-inflation category includes only four countries, either because these countries are unique (Zaire, Israel) or because the data series for other high-inflation countries are not available for the entire period 1963–1984. Moreover, criterion (iii) excludes all other countries (except U.S., U.K. and France) in the G7 group, since z is far from zero (for example, for West Germany and Japan, it is 0.75 and 0.78, respectively.) The results for France are reported in footnote 20 below.

¹⁶For a comprehensive discussion on various types of velocity, that is, income, transaction, and financial, using M1 and the monetary base, see Osborne (1986).

TABLE 2. *Summary of Indicators: 1963–1985*

Country	Per capita GNP in 1986 (U.S.\$)	Average Inflation Rate (%)	Average Output Growth Rate (%)	Average Money Growth Rate (%)	Average Money Velocity
Zaire	160	27.69	2.43	29.55	6.91 (0.34)
Madagascar	230	9.61	0.97	9.02	10.77 (0.31)
Ghana	390	25.10	0.58	25.43	8.67 (0.45)
Haiti	330	6.58	2.15	11.63	9.43 (0.65)
India	290	7.85	3.94	12.05	8.41 (0.16)
Bolivia	600	55.92	3.36	57.83	8.62 (0.32)
Philippines	560	11.45	4.12	14.40	14.39 (0.33)
Turkey	1110	22.55	4.81	27.39	7.75 (0.21)
Morocco	590	6.50	4.01	11.38	7.11 (0.15)
Thailand	810	5.38	6.90	10.61	10.25 (0.29)
Brazil	1810	47.32	6.58	46.96	15.16 (1.30)
Portugal	2250	12.11	4.51	14.29	4.02 (0.14)
Mexico	1860	20.93	3.86	33.01	9.81 (0.84)
Venezuela	2920	7.85	3.28	14.43	10.15 (0.54)
S. Africa	1850	9.64	3.74	11.06	18.17 (0.62)
Israel	6210	42.35	5.22	42.01	1.27 (0.16)
Ireland	5070	10.26	3.77	12.12	7.06 (0.30)
Spain	4860	11.42	3.92	17.17	7.29 (0.26)
U.S.	17480	5.66	2.91	6.47	14.42 (0.49)
U.K.	8870	8.90	2.23	6.55	15.00 (1.20)

NOTES: (a) The numbers in parentheses are standard deviations of means of average money velocity. (b) Growth rates are computed as the averages of the corresponding annualized rates. (c) Per capita GNP data are taken from Stern (1989); all other data are computed from various issues of the *International Financial Statistics*.

$$\ln VM(t) = a_0 + a_1 GRM(t) + a_2 \ln RGDP(t) \quad (30)$$

for each country. The theory described predicts $a_1 < 0$, at least when $\theta_\pi \geq 0$. The sign of a_2 captures the effects of ϕ and A discussed in Section 3, and is, in general, ambiguous.

To avoid the problems of multicollinearity between the two independent variables and of the endogeneity of real income, we lag real GDP by one period. In the cases where simple OLS estimation exhibits serious autocorrelation, the Cochrane-Orcutt iterative procedure is adopted. We also test for the integration of the series.¹⁷ In eleven cases the estimated coefficient of the first-

¹⁷Velocity can also depend on many institutional factors, such as the required reserve ratio, the structure of financial intermediation procedures, etc. Nevertheless, consistent data series on these variables are not available.

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order autocorrelation process is not found to be statistically less than one; thus the study of the first differences of the series is essential.¹⁸ The results are summarized in Table 3.

By examining the regression estimates, we find that for thir-

¹⁸The relatively small number of observations does not allow for standard unit root tests. Instead, a *t*-test on the estimated coefficient of first-order autocorrelation is used.

TABLE 3. *Regression Results: 1963-1985*

Country	Estimate (<i>t</i> -value) of coefficient of		<i>RHO</i>
	<i>GRM</i>	<i>RGDP</i>	
<i>Zaire</i> ¹	-0.51 (-3.16)**	0.29 (1.27)	NA
<i>Madagascar</i> ³	-0.58 (-4.12)**	-0.27 (-0.55)	NA
<i>Ghana</i> ³	-0.13 (-0.84)	-0.02 (-0.06)	NA
<i>Haiti</i> ²	-0.19 (-1.13)	-1.38 (-7.10)**	0.503 (2.34)
<i>India</i> ²	-0.61 (-4.07)**	-0.43 (-3.55)**	0.771 (5.93)
<i>Bolivia</i> ¹	0.03 (0.81)	0.04 (0.28)	NA
<i>Philippines</i> ²	-0.67 (-5.34)**	0.29 (2.14)**	0.670 (3.57)
<i>Turkey</i> ³	-0.42 (-1.78)*	-0.07 (-0.25)	NA
<i>Morocco</i> ¹	-0.75 (-4.50)**	-0.24 (-7.85)**	NA
<i>Thailand</i> ¹	-0.51 (-1.78)*	0.28 (13.05)**	NA
<i>Brazil</i> ³	-0.23 (-1.85)*	0.63 (2.19)**	NA
<i>Portugal</i> ³	-0.66 (-2.80)**	0.32 (0.88)	NA
<i>Mexico</i> ³	-0.28 (-3.62)**	-0.28 (-1.56)	NA
<i>Venezuela</i> ³	-0.23 (-0.90)	-0.49 (-1.04)	NA
<i>S. Africa</i> ¹	-0.31 (-2.40)**	0.59 (10.91)**	NA
<i>Israel</i> ³	-0.42 (-3.02)**	0.61 (0.99)	NA
<i>Ireland</i> ³	-0.53 (-5.22)**	0.06 (0.17)	NA
<i>Spain</i> ³	-0.49 (-4.16)**	-0.07 (-0.16)	NA
<i>U.S.</i> ²	-0.63 (-2.45)**	0.97 (8.49)**	0.514 (2.86)
<i>U.K.</i> ³	-0.43 (-2.71)**	0.62 (1.07)	NA

NOTES: (a) *VM*, *GRM*, and *RGDP* are money velocity, money growth rate, and real *GDP*, respectively. (b) 1 indicates simple OLS estimation using the logarithms of the levels, while 2 indicates estimation using the Cochrane-Orcutt method for the correction of autocorrelation. Finally, 3 indicates OLS estimation using the logarithms of the first differences. (c) *RHO* is the Cochrane-Orcutt estimated coefficient of the first-order autocorrelation (N.A. indicates not applicable, meaning that *RHO* is statistically neither different from zero nor less than one, at the 5% significance level). (d) ** and * indicate the significance of the explanatory power of the independent variable at the 5% and 10% levels, respectively. The associated number of degrees of freedom is 16.

teen (sixteen) out of twenty countries the effects of money growth on money velocity are significantly negative at the 5% (10%) level.¹⁹ The effects of real income, on the other hand, are not so significant, especially after correcting the autocorrelation problem. This could be the case because of the following reasons: (i) as we showed in Section 4, an increase in the technological parameter, A , will have two offsetting effects on money velocity; and, (ii) the level of GDP may capture both credit enhancement policies and financial development programs that encourage the use of money. We have also examined the correlation between real money balances and the money growth rate. In most cases, we have found the correlation coefficient to be non-negative, which, in terms of our model, is consistent with $\theta_\pi > 0$.

Simple Cross-Country Evidence

To strengthen the validity of our main finding, we next plot the average money velocity measures against average money growth

¹⁹Since the velocity of money is not allowed to have a deterministic trend (see Equation [30]), the constant term in the first-difference estimations is suppressed. A formal test on whether velocity is integrated or trend-stationary cannot be performed due to the small sample. Nevertheless, in the case of the U.S., using quarterly data, we have found that velocity has a stochastic trend (it is integrated of order one).

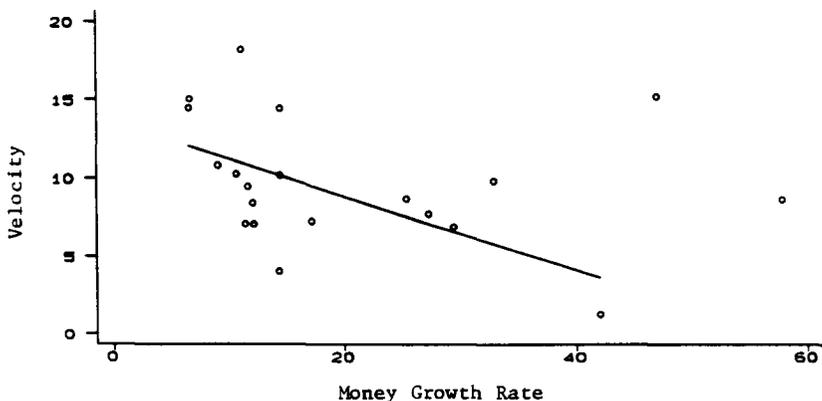


Figure 1.
Money Growth and Velocity: 20-Country Data (1963–1985)

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rate for all countries, as shown in Figure 1.²⁰ Excluding Bolivia and Brazil, which are the two countries with the highest inflation rate in the sample, one can see the negative relation between the two variables. That is, for countries with higher money growth, their velocities of money tend to be lower. The overall correlation coefficient is -0.26 , but if we exclude Bolivia and Brazil from the sample it becomes -0.58 . The same result obtains even if we replace the money growth rate by the inflation rate (the correlation coefficient is -0.21 and if we exclude Bolivia and Brazil it becomes -0.55).

In order to examine how sensitive the result is to the choice of the sample period, we next divide the entire period 1963–1985 into two sub-periods, 1963–1973 and 1974–1985. The results can be found in Figures 2a and 2b, respectively. For the first (second) period, the correlation coefficient between the money growth rate and velocity is -0.13 (-0.19) but if we exclude Bolivia and Mexico (Bolivia and Brazil), the first two countries with the highest inflation rate, then it becomes -0.60 (-0.61). Thus, the negative relationship between the money growth rate and the money velocity appears to be robust.²¹

²⁰In the case of France, the regression coefficients for *GRM* and *RGDP* (in the first-difference specification) are found to be -0.47 (-3.70) and 0.65 (1.62), respectively, with *t*-values in parentheses.

²¹It should be noted that any attempt to pool the data is rejected by various *F*-tests. That is, all the coefficients vary significantly across countries.



Figure 2a.
Money Growth and Velocity: 20-Country Data (1963–1973)

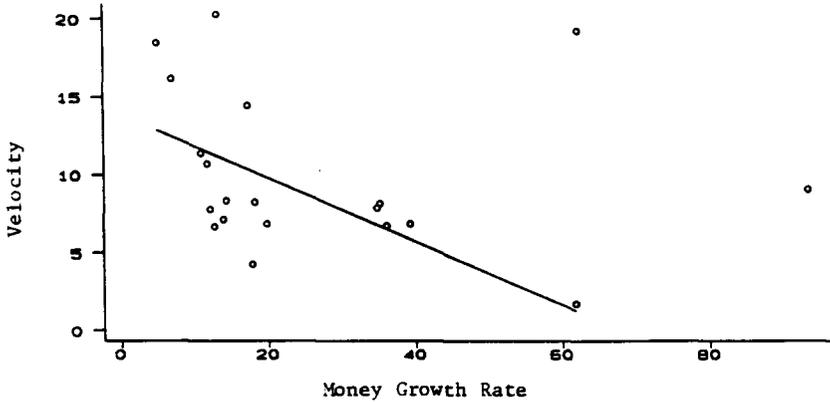


Figure 2b.
Money Growth and Velocity: 20-Country Data (1974-1985)

6. Concluding Remarks

We develop a modified cash-in-advance model in which money is required prior to purchases of the consumption good and prior to a fraction of capital good. We find that higher money growth equilibria are associated with lower welfare and lower velocity than low money growth equilibria. First, a higher money growth rate generates a reverse Tobin effect that leads to a lower capital stock. Under the assumption of decreasing returns to scale this will result in a reduction of money velocity. Furthermore, if we allow the fraction of capital good subject to the liquidity constraint to depend positively on the inflation rate, then there will be an additional downward pressure on money velocity. Such a theoretical finding is supported by empirical evidence.

The reader should nevertheless be warned that our model cannot be applied to the case of hyperinflation without further generalization. To see this, suppose that the transactions frequency, γ , is allowed to change over time. This transactions frequency can be regarded as the inverse of the optimal time between trips to the bank in a generalized inventory model as described in Romer (1986). One may then rewrite the cash-in-advance constraint, (3), as

$$c_t + \theta(\pi_t, \phi_t)k_{t+1} = \gamma(\pi_t) \left(\frac{m_t}{1 + \pi_t} + \tau_t \right),$$

where $0 < \gamma < \infty$ and $\gamma_\pi > 0$, indicating higher transactions frequency resulting from a higher inflation rate. It can be shown that,

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with this modification, the effect of a higher money growth rate on the velocity of money becomes ambiguous. Note however that the countries we selected for our empirical study have not reached the stage of hyperinflation and hence the transactions frequency effect is not expected to be very influential.²²

In this paper we have assumed the presence of decreasing returns to scale with respect to per capita capital. Recently, there has been an interest in models of constant (Rebelo 1991) or increasing returns to scale (Romer 1986). In these models changes in the money growth rate will affect not only the level but also the growth rate of output. We believe that it would be interesting to use the framework developed here and examine the effects of money growth on velocity in the presence of a non-decreasing returns to scale technology. We leave this, however, as a topic for future research.

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²²One might tend to think that there is a bias introduced by our selection criteria. This could be the case if technical progress, population growth, and/or the transactions frequency play an essential role. Nevertheless, we believe that by selecting countries from different income and inflation groups the effect of such a possible bias has been minimized.

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Appendix

List of Symbols

- A = Technological parameter.
 c = Per capita consumption.
 f = Production function.
 i = Nominal interest rate.
 k = Per capita capital.
 M = Nominal money holdings in the beginning of a period.
 m = Real money holdings in the beginning of a period.
 P = Price level.
 r = Real interest rate.
 t = Time index.
 U = Lifetime utility function.
 u = Felicity function.
 V = Velocity of money.
 W = Value function.
 Y = Per capita output.
 GRM = Growth rate of the monetary base.
 MPK = Marginal product of capital.
 $RGDP$ = Real GDP.
 VM = Money velocity measured as nominal GDP/monetary base.

β = Discount rate.

γ = Transactions frequency.

θ = Fraction of the capital good subject to C-I-A constraint.

μ = Money growth rate.

π = Inflation rate.

τ = Lump sum cash transfer.

ϕ = Parameter measuring credit enhancement.

I_j = Partial derivative of I with respect to its j th argument.