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The economics of 'new blood' ☆

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Abstract

We construct a dynamic general-equilibrium model of search and matching where public knowledge grows through time and workers accumulate a fraction of this knowledge through education/retraining. Due to search delays, the unemployment pool is populated by vintages of workers of differing productivities. Through intergenerational rivalry, the human capital of older generations is rendered obsolete relative to that of the new blood. Higher knowledge growth exacerbates intergenerational competition, thereby lowering education and growth while raising unemployment and inequality. These findings help explain wage compression/expansion and the hump-shaped wage-tenure profile across cohorts.

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Nomenclature

English Alphabet (lower case)

a_0	value of hiring a worker (or worker productivity)
a_H	high-type worker productivity
a_L	low-type worker productivity
c	schooling cost
e	effective labor units
h	time increment
k	private stock of human capital
m	flow matches
m_0	matching parameter
$\tilde{m}_0$	a critical value for m_0
q	experience adjusted vintage
$\hat{q}^U$	critical point of q at which an unemployed worker retrains himself
$\hat{q}^E$	critical point of q at which an employed worker is retrained at a given firm
$\hat{q}_0^E$	critical point of q at which a newly employed worker is first retrained
r	subjective discount rate
s	schooling effort (or education)
$\bar{s}$	society's average level of education
t	time index
t^b	birth date
t'	employment date
w	capitalized value of wage in effective units
u, v, z	integration dummies
$g(q)_U$	steady-state density function of unmatched type-q workers
$g(q)_E$	steady-state density function of matched type-q workers
$z(q)$	defined as $a_0 \exp[-q]$

(upper case)

A_i	transformed constants in various parameters ($i = 1, 2, 3, 4, 5, 6$)
B_i	transformed constants in various parameters ($i = 1, 2, 3, 4, 5, 6, 7, 8, 9$)
C_0	retraining cost
E	mass of employed workers
H	vintage human capital
K	public knowledge
K_0	initial stock of public knowledge
M	matching function
Q	terminal value of q
ΔQ	defined as Q-q
Q_E	critical point of q at which firm retrains its worker
T	terminal search horizon

U	mass of unmatched workers
V	mass of vacancies
W_0	transformed constant in various parameters
$\tilde{Y}$	flow output
Y	flow output in effective units
$G(q)_{\mathrm{U}}$	steady-state cumulative distribution function of unmatched type-q workers
$G(q)_{\mathrm{E}}$	steady-state cumulative distribution function of matched type-q workers
$F(q)_{\mathrm{U}}$	steady-state cumulative probability-distribution function of unmatched type-q workers
$F(q)_{\mathrm{E}}$	steady-state cumulative probability-distribution function of matched type-q workers
$\tilde{J}(t,q)_{\mathrm{U}}$	type-q worker's unmatched value at date t
$J(t,q)_{\mathrm{U}}$	type-q worker's unmatched value at date t in effective units
$\tilde{J}(t,q)_{\mathrm{E}}$	type-q worker's matched value at date t
$J(t,q)_{\mathrm{E}}$	type-q worker's matched value at date t in effective units
$S(q,I)$	surplus generated from a match with a type-q worker ($I=0$ or 1)
$\tilde{V}(v;t',q)$	type-q worker's valuation hired by firm v at employment date t'
$V(v,q)$	type-q worker's valuation hired by firm v in effective units
$\tilde{W}(t',q)$	flow wage of a type-q worker employed at t'
$W(q)$	flow wage of a type-q worker in effective units

Greek (lower case)

β	birth and death rate
β'	defined as $\beta+r+\delta$
$\tilde{\beta}$	defined as $\beta+r-\gamma_{\mathrm{G}}$
γ_{G}	exogenous public knowledge growth rate
γ_{E}	on-the-job learning rate
γ_{U}	human capital depreciation rate during unemployment
δ	flow job separation rate
$\hat{\delta}$	defined as $\delta/(\tilde{\beta}+\delta)$
λ_i	characteristic roots
μ	flow worker contact rate
$\bar{\mu}$	upper bound of μ
η	flow vacancy contact rate
$\bar{\eta}$	upper bound of η
η^{EE}	flow vacancy contact rate function under equilibrium entry
η^{SS}	flow vacancy contact rate function under steady-state matching
v_0	fixed entry cost
τ	vintage (or total labor market experience)
τ_{E}	employment tenure
τ_{U}	unemployment duration
ϕ	fraction of knowledge acquired by individual

$\bar{\phi}$	upper bound of ϕ
ξ	an elasticity defined as $-(\mu/\eta)(d\eta^{SS}/d\mu)$
$\tau(q)_M$	maximum tenure of a type-q worker at a given firm
$\chi(\mu)$	transformed function of μ
(upper case)	
Δ_i	transformed constants in various parameters ($i = 1, 2$)
Γ_E	on-the-job-learning function
$\hat{\Pi}_F$	expected value of filling a vacancy
$\tilde{\Pi}(t)_V$	value of unfilled vacancy at date t
$\Pi(t)_V$	value of unfilled vacancy at date t in effective units
$\tilde{\Pi}(t,q)_F$	value of filled vacancy with type-q worker at date t
$\Pi(t,q)_F$	value of filled vacancy with type-q worker at date t in effective units

New blood: "new members or elements, with new ideas and experiences, admitted to a society or organization." [Oxford English Dictionary 1999, Second Edition.]

1. Introduction

Employers are often keen to fill vacant posts with 'new blood,' reflecting the 'vitality' and 'vigor' that such workers are capable of bringing to an organization. In this paper we posit and study a mechanism wherein the cutting edge of new blood workers stems from economic growth. To this end, we consider an environment in which: (i) the stock of productive public knowledge grows through time and (ii) new entrant workers acquire a proportion of this knowledge through their educational efforts while young. Once their schooling is complete, workers enter the primary labor market and search for employment. The time consuming nature of search, random job break ups, and the individual embodiment of human capital imply that, at any given point in time, the labor market is populated by overlapping generations ('vintages') of workers of differing productivities. The members of current generations of workers, by virtue of their recent acquaintance with the 'state of the art' knowledge, enter the labor market with a productive edge relative to previous entrants.

As a practical illustration of this phenomenon, consider the experience of a typical high school graduate: today such a student might be expected to have a basic working knowledge of computers; no such expectation would be held for a similar student of even two decades ago. Similar patterns of knowledge growth and human capital obsolescence may be observed in diverse professional occupations, such as surgery, where sole reliance on a steady hand has given way to the use of the laser, or in basic research, where today's cutting edge work often ends up in the form of routine exercises solved by tomorrow's crop of graduate students.

Of course, a more mundane explanation of this phenomena is that after reaching some critical point, workers' human capital levels relentlessly decline with age. This view is clearly relevant for some occupations, such as professional sports and certain types of manual labor, where strength and ability are critical. Yet, it is less obviously pertinent for others, such as the legal, management, information technology, medical, and other research/technical professions, in which real wages and arguably productivity often (weakly) increase with labor market experience. It is for these latter occupations, in which knowledge growth is important, that our framework is intended.[1]

We construct a dynamic general-equilibrium model that encompass the emergence of the new blood phenomenon and study its labor market consequences. The framework we propose is one in which knowledge growth and its subsequent embodiment in new entrant workers gives them their competitive edge.[2] This structure leads to a form of intergenerational rivalry in which the human capital of older generations is rendered obsolete relative to that of younger ones.[3] In contrast to the recently developed endogenous growth literature (emphasizing the importance of human capital for growth), our approach focuses upon the reverse effect of growth on human capital acquisition and labor market efficacy.

We show that the new blood effect may have profound implications for labor market behavior. These effects include: (i) changes in the life-cycle wage pattern for individual workers, (ii) an explanation for the Mincer hump-shaped earnings-tenure profile observed in cross-section studies, and (iii) the compression and expansion of the aggregate wage distribution observed in time series data.[4] Also, as a corollary of our approach, we are able to offer novel insights into the effect of growth on unemployment.

[1] Neuman and Weiss [28] distinguish between two types of human capital depreciation: internal and external. The first type is the depreciation attributable to the worker, e.g., loss of physical ability and mental capacity due to a worker's aging. The second type is the depreciation caused by external factors, e.g., loss of market value due to changes in the economic environment, such as obsolescence of a worker's education. Using data from the Israeli Census, they find strong evidence for the presence of external depreciation, especially in high-tech oriented industries. The new blood effect identified in this paper coincides precisely with their concept of external depreciation.

[2] Following Stokey [37] and Laing, Palivos and Wang [19] we explicitly distinguish between the stock of knowledge available to society as a whole and that which is possessed by any particular individual in it.

[3] An elementary (but important) point is that small differences in growth rates can have profound effects on levels. For instance, suppose the productivity of each generation is simply proportional to the present stock of public knowledge and knowledge grows at a constant rate. Then, with a growth rate of 1% it would take about 69 years before new entrant workers became twice as productive as the current generation. Alternatively, if the growth rate is 10% it would take only 6.9 years.

[4] It is a well-known empirical fact that real earnings-tenure profiles are 'hump-shaped' (see, for example, Mincer [23], Murphy and Welch [27], and Neuman and Weiss [28]). Also, wage income disparities by cohort or by tenure were narrowed in the 1940s and sharply widened in the 1960s and 1980s. These phenomena are, respectively, referred to as wage compression and wage expansion—see Goldin and Margo [15], Solon et al. [36], and Murphy and Welch [27], respectively, for the 1940, 1960 and 1980 episodes.

The paper is structured as follows. Section 2 describes the environment, and Section 3 presents the key steady-state relationships underpinning the model. For ease of presentation, Sections 4 and 5 consider a simple version of the model in which: (i) the only source of growth is an exogenous increase in the stock of public knowledge (in particular, we rule out on-the-job learning) and (ii) there is no loss of skill during unemployment. Section 4 derives and studies the properties of the equilibrium wage agreement, while Section 5 characterizes the general equilibrium properties of the model. Section 6 considers endogenous education and admits worker re-training. Section 7 analyzes several extensions of the model, including more realistic versions, which relax the restrictions imposed earlier, as well as a directed-search variant in which the wage fulfills its familiar allocative role.[5] Finally, Section 8 offers some concluding remarks.

1.1. Related literature

Chari and Hopenhayn [8] present an overlapping-generations model with on-going technological improvement and investment in technology-specific human capital. As a result of these specificities, they are able to establish technological diffusion in which the peak use of a new technology occurs some time after its initial introduction. However, in their model workers live for but two periods and possess skills in only one of them. As a consequence, their paper cannot address the new blood effect, which is the focus of this paper.

There are some related papers studying how technical progress may affect labor market outcomes in the presence of search frictions. In Aghion and Howitt [4], technological advancement leads to the creation of new jobs (by stimulating entry through raising the returns from production) as well as the destruction of jobs (in obsolete sectors). They show that either effect may dominate and that the unemployment rate need not be monotone in the innovation rate. Mortensen and Pissarides [26] consider destruction and creation of jobs over the business cycle and calibrate the creative destruction dynamics. An important distinction between this paper and theirs is that whereas they focus on the creative destructive effects of new technologies on unemployment, we consider the consequences of intergenerational rivalry and skill obsolescence on human capital accumulation and income distribution.

Laing et al. [19] construct a search model that examines the relationship between trade frictions and both the level of education undertaken by workers and the rate of on-the-job learning. However, although that paper includes education and search, it differs substantially in both structure and focus from this paper. In particular, since in that paper there is no public knowledge growth there is neither the possibility of intergenerational competition nor that of a 'new blood' effect. These latter two features are the quintessence of the current paper.

[5] Our basic search and bargaining framework is of the sort considered in the pioneering papers of Diamond [13], Mortensen [25] and Pissarides [31]. Our directed search model builds upon Moen [24].

2. The basic framework

We construct a search model incorporating knowledge growth and human capital obsolescence. Time is continuous, beginning at date $t = 0$. The economy is populated by a continuum of workers, with mass normalized to unity, and a continuum of firms, whose population is determined as part of the equilibrium. All agents are risk neutral and discount the future at the common rate $r > 0$.

There are three separate theaters of economic activity, corresponding to an educational sector, a primary 'search' labor market, and a competitive secondary sector (which can also be interpreted either as 'long-term' unemployment or as 'home production').

2.1. Workers and firms

Workers are born and die at the rate β. Immediately after their birth they enter the educational sector and accumulate human capital. We assume that educational effort, s, is costly and, for simplicity, instantaneous. Once their education is complete, workers enter the primary-labor market in search of employment. Each worker is endowed with a unit of labor, which is supplied inelastically without disutility from effort.

The primary sector is populated by many firms, each of which possesses a fixed-coefficient production technology capable of utilizing the labor services of at most one worker. We assume, through a suitable choice of units, that, at each point in time, the flow output, $\tilde{Y}(t)$, produced by a worker equals his stock of human capital $H(t)$. In this sector, unemployed workers and unfilled jobs are brought together through a stochastic matching technology.[6] The instant a vacancy and a worker make contact, they bargain over the division of any surplus. If an agreement is reached production commences immediately.

We assume that vacancies are completely durable and are homogeneous in every respect. In the event of an employee's death, firms re-enter the labor market and search for new employees. In addition, we assume that matches dissolve at the constant rate $\delta \geqslant 0$. This formulation leads to the subsequent re-entry of workers into the primary labor market, which enables us to study workers' retraining decisions in the presence of human capital obsolescence. We model the secondary sector parsimoniously. There workers can instantly locate employment at a competitive flow wage normalized to zero.

2.2. Human capital

As in Stokey [37], we distinguish between the stock of public knowledge, $K(t)$, and the quantity of human capital $k(t)$ acquired by a given worker through his educational efforts. With regard to $K(t)$, we assume that research and development activities lead to innovations and discoveries that continually add to the economy's stock of public knowledge, causing it to grow at the exogenous rate $\gamma_G > 0$ (we discuss

[6] In Section 7.3, we consider a directed-search approach to our model.

the welfare implications of endogenizing γ_G in Section 6.1.2). This yields,

$$K(t) = K_0 \exp[\gamma_G t] \quad K_0 > 0, \tag{2.1}$$

where K_0 is the initial stock of knowledge. Increases in K_0 are used to model the effect of a once off increase in knowledge resulting, for example, from a major discovery or innovation. Educational effort, s, enables workers to absorb a proportion of the available stock of knowledge, $K(t)$, which raises their productivity and their wage income once employed. Consider:

Assumption 1. *An individual born at date t, who exerts initial schooling effort s, accumulates a private stock of human capital k, given by*

$$k(s,t) = \phi(s)K(t). \tag{2.2}$$

The function $\phi : \mathbb{R}_+ \to [0,1]$ is strictly increasing, strictly concave, and twice continuously differentiable, satisfying: $\phi(0) > 0$ and $\lim_{s\to\infty} \phi(s) \leqslant \bar{\phi} < 1$.

Notice in (2.2) that an individual's success in accumulating human capital depends not only upon his schooling effort, s, but also upon the total stock of knowledge, $K(t)$, available to him at the date his education takes place. This is the essence of the *new blood effect*: for any given level of schooling effort, s, future generations of workers, by virtue of the growth of $K(t)$, possess a productive edge relative to the members of earlier generations. According to Assumption 1 there are diminishing returns to investment in education. The boundary conditions, $\phi(0) > 0$ and $\bar{\phi} < 1$ ensure, respectively, that, regardless of s, workers always possess some human capital and that workers are never omniscient.

Consider a point in time t and a worker born at some previous date $t^b \leqslant t$. At date t this worker's total labor market experience, or '*vintage*,' τ, is simply: $\tau \equiv \tau_E + \tau_U = t - t^b$, where τ_E is the worker's accumulated employment experience and τ_U is the total time he has spent unemployed. The pair (τ_E, τ_U) is the worker's realized labor market history at that point. Although much of our analysis focuses squarely on the effects of intergenerational rivalry, in setting up the model it is useful to consider a more general environment that allows for the accumulation of skills on the job and for the loss of skills while unemployed.[7] Assumption 2 describes the evolution of worker human capital with respect to (τ_E, τ_U).

Assumption 2. (i) *Workers acquire general human capital at the constant rate $\gamma_E \geqslant 0$ through on-the-job learning*; (ii) *during unemployment workers' skills atrophy at the constant rate $\gamma_U \geqslant 0$.*

[7] Several authors have argued that one of the key problems associated with protracted spells of unemployment is that during such times workers' skills may decline to the point of obsolescence. Phelps [30] and Layard et al. [21] consider this possibility, whereas Jacobsen et al. [17] and Ruhm [35] provide empirical evidence of earnings losses of displaced workers. Pissarides [32] develops a model with a loss of skill during unemployment and shows that, in such an environment, the effects of a temporary contractionary shock can persist for a long time. Coles and Masters [10] construct a model of optimal unemployment insurance with skill loss.

The assumption, in part (i), that only *general* human capital is acquired on the job is inconsequential, since only general human capital is transferred between matches and all of the results of interest in our paper occur as workers make the transition into and out of unemployment.[8] Assumption 2 implies that the date t human capital possessed by a vintage τ worker with history (τ_E, τ_U) is given by

$$H(t, s, \tau_E, \tau_U) = k(s, t - \tau)\exp[\gamma_E\tau_E - \gamma_U\tau_U], \tag{2.3}$$

where $t - \tau$ is the date the worker completed his education. Using (2.1) and (2.2) in (2.3) gives,

$$H(t, s, \tau_E, \tau_U) = k(s, t)\exp[-(\gamma_G - \gamma_E)\tau_E - (\gamma_U + \gamma_G)\tau_U]. \tag{2.4}$$

Notice in (2.4) that the new blood effect, operating through γ_G, re-enforces any loss of human capital that workers may experience during unemployment. The new blood effect is also quite distinct from this latter phenomenon, since, as (2.4) makes clear, it also operates while the worker is employed. In the subsequent analysis considerable use is made of,

$$q = q(\tau_E, \tau_U) \equiv (\gamma_G - \gamma_E)\tau_E + (\gamma_U + \gamma_G)\tau_U. \tag{2.5}$$

Inspection of (2.4) reveals that $q(\tau_E, \tau_U)$ characterizes the human capital possessed by a vintage τ-worker (with history (τ_E, τ_U)) *relative to* that possessed by a worker from the most recent generation $(k(s, t))$. Notice, if $q = 0$ the worker's human capital *is* $k(s, t)$, which is identical to that possessed by the newest crop of entrants. In contrast, if $q > 0$ $(q < 0)$ the worker has faced a relative *decline* (*increase*) in human capital. The function $q(\cdot)$ is consequently a mapping from each worker's realized labor-market history (τ_E, τ_U) into a variable q that determines his relative-to-new-entrant human capital. In what follows we call q the worker's *experience adjusted vintage* and, for brevity, often refer to it simply as either the worker's *adjusted vintage* or his *type*. The convention that a larger value of q represents a relative decline in human capital is quite natural, since, in this paper, it is workers from older vintages who suffer from human capital obsolescence. Given the earlier normalization that flow output, $\tilde{Y}(t)$, produced by a worker equals his human capital stock, $H(t)$, we can use (2.4) and (2.5) to write,

$$\tilde{Y}(t) = \tilde{Y}(t, s, q) = k(s, t)\exp[-q]. \tag{2.6}$$

Our analysis of the steady-state properties of the model is facilitated by measuring all of the variables in terms of effective-labor units, defined by: $e(t) \equiv \exp[\gamma_G t]$. Thus, flow output per effective labor unit, Y, equals:

$$Y(s, q) \equiv \tilde{Y}(t, s, q)/e(t) = K_0\phi(s)\exp[-q]. \tag{2.7}$$

Notice from (2.7) that conditional upon a given level of education, the worker's q-value uniquely determines his flow output. Thus, the worker's type q embodies all of the economically relevant information about his labor market history, (τ_E, τ_U) at that point.

[8] In Section 7.2, we allow the rate of on-the-job learning, γ_E, to depend upon the length of the worker's tenure at the firm.

2.3. Entry

We assume that firms incur a constant cost, v_0, (in effective units) each time they enter the labor market.[9] The entry cost v_0 possesses a variety of interpretations, including the (irreversible) cost of a unit of capital as well as the cost of advertising the vacancy. We assume unrestricted entry of firms into the search market, so that any number of firms can, upon paying the entry fee v_0, (instantly) move into the primary sector should it prove profitable for them to do so. In practice, the cost of a unit of capital depends upon a host of factors, such as the availability of credit and the economy's tax structure; the latter is often geared toward providing investment incentives. We impose the following condition:

Condition N. (i) $\int_0^\infty \exp[-(r+\beta+\delta-(\gamma_G+\gamma_E))u]\,Y(0,0)\,du > v_0$; (ii) $\beta + r > \gamma_G + \gamma_E$.

Part (i) of Condition N ensures a non-degenerate equilibrium, since firms enter the primary labor market regardless of workers' initial schooling effort, s. Part (ii) requires that the growth-adjusted discount rate is always positive, so that all expected-discounted valuations are bounded above.

2.4. Matching

Let $U(V)$ denote the mass of searching workers (vacancies) in the primary sector. The flow probability that a worker (vacancy) locates a vacancy (worker) is denoted $\mu(\eta)$. Since vacancies are filled by one (and only one) worker, it follows that:

$$\mu U = \eta V. \tag{2.8}$$

Although μ and η are determined in equilibrium, both workers and vacancies treat them as parametric when making their decisions. The search environment is characterized by a *matching technology*, $m = m_0 M(U, V)$, which describes the instantaneous flow meeting rate between unfilled vacancies and searching workers.

Assumption 3. *The matching function* $M : \mathbb{R}_+^2 \to \mathbb{R}_+$, (i) *is twice continuously differentiable*, *strictly increasing and strictly concave in each argument*, (ii) *exhibits constant returns to scale and* (iii) *satisfies the Inada conditions*: $\lim_{j\to 0} M_j = \infty$, $\lim_{j\to\infty} M_j = 0$, $j = U, V$, *and the boundary conditions*: $M(0, V) = M(U, 0) = 0$.

An increase in the number of participants on either side of the market raises the instantaneous level of matches, but at a diminishing rate [part (i)]. The Inada

[9] The main defense of the first part of this assumption is that (i) this formulation is necessary for balanced growth (see, for example, Aghion and Howitt [3]) and (ii) allowing for general time-dependent formulation quickly leads to an intractible structure. The second part of the assumption, that v_0 is incurred each time the firm (re-)enters the labor market is made purely for simplicity and, indeed, is relaxed in Section 7.2. We remark that admitting a *flow* search cost v_1 in addition to v_0 does not alter the main findings of the paper.

conditions ensure an interior steady-state solution. The extent of the search frictions in the primary market is conveniently parameterized by $m_0 > 0$. An increase in m_0 raises—for given U and V—the instantaneous matching rate and therefore reflects a reduction in the severity of search frictions. In practice, the severity of market frictions will depend upon factors such as the quality of the economy's infrastructure.

2.5. Asset value functions

At any point in time a worker is either employed (E) or is unemployed (U). Likewise, a firm is either currently employing a worker (F) or else is looking for one (V). Define, $\tilde{J}(t,q)_{\mathrm{E}}$ as the value to a type q-worker of accepting employment at date t. Likewise, let $\tilde{J}(t,q)_{\mathrm{U}}$ represent the value to the worker of remaining unemployed at that time. Similarly, $\tilde{\Pi}(t,q)_{\mathrm{F}}$ and $\tilde{\Pi}(t)_{\mathrm{V}}$ represent the respective values to a firm of filling a vacancy at date t with a type q worker or holding open the vacancy. Our analysis of the steady state is facilitated by measuring all variables in terms of effective-labor units, $e(t) \equiv \exp[\gamma_{\mathrm{G}} t]$. Accordingly, define: $J(q)_{\mathrm{E}} \equiv \tilde{J}(t,q)_{\mathrm{E}}/e(t)$, and $J(q)_{\mathrm{U}} \equiv \tilde{J}(t,q)_{\mathrm{U}}/e(t)$. Likewise, let: $\Pi(q) \equiv \tilde{\Pi}(q)_{\mathrm{F}}/e(t)$ and $\Pi_{\mathrm{V}} \equiv \tilde{\Pi}(t)_{\mathrm{V}}/e(t)$. Consider:

Proposition 1 (Asset values). *Under Condition N, the asset value functions for each $q \leqslant Q$ are:*

$$J(q)_{\mathrm{E}} = w(q) + \frac{\delta}{(\gamma_{\mathrm{G}} - \gamma_{\mathrm{E}})} \int_q^Q \exp\left[-\frac{\beta + r + \delta - \gamma_{\mathrm{G}}}{\gamma_{\mathrm{G}} - \gamma_{\mathrm{E}}}(v - q)\right] J(v)_{\mathrm{U}}\, dv \quad \textit{if } \gamma_{\mathrm{E}} \neq \gamma_{\mathrm{G}}, \tag{2.9}$$

$$J(q)_{\mathrm{E}} = w(q) + \frac{\delta}{\beta + r + \delta - \gamma_{\mathrm{G}}} J(q)_{\mathrm{U}} \quad \textit{if } \gamma_{\mathrm{E}} = \gamma_{\mathrm{G}}, \tag{2.10}$$

$$J(q)_{\mathrm{U}} = \frac{\mu}{\gamma_{\mathrm{G}} + \gamma_{\mathrm{U}}} \int_q^Q \exp\left[-\frac{\beta + r + \mu - \gamma_{\mathrm{G}}}{\gamma_{\mathrm{G}} + \gamma_{\mathrm{U}}}(v - q)\right] J(v)_{\mathrm{E}}\, dv, \tag{2.11}$$

$$\Pi(q)_{\mathrm{F}} = a_0 \exp[-q] - w(q), \tag{2.12}$$

$$\Pi_{\mathrm{V}} = \frac{\eta}{\eta + r - \gamma_{\mathrm{G}}} \hat{\Pi}_{\mathrm{F}}, \tag{2.13}$$

where $a_0 \equiv a(s, K_0) = \frac{\phi(s)K_0}{(r+\beta+\delta-\gamma_{\mathrm{E}})}$, Q is a limit of integration (determined below), and $\hat{\Pi}_{\mathrm{F}} \equiv E\{\Pi(q)_{\mathrm{F}}\}$ is the expected value of filling a vacancy with a worker drawn at random from the pool of unemployed job seekers.

In (2.9), (2.10) and (2.12) the term $w(q)$ is the expected discounted value of wage income received by the worker (measured in effective units). In what follows we refer to $w(q)$ simply as 'the wage' and assume that at each point in time a worker's flow wage income, W, is proportional to his current stock of human capital.[10] In (2.12) the term $a_0(s, K_0)\exp[-q]$ equals the expected discounted revenues accruing to the firm from hiring a type q worker. It follows that $a_0 \equiv a(s, K_0)$ is the value of hiring a worker who belongs to the most recent generation.

The asset-value functions possess straightforward interpretations. For example in (2.9) the value to a worker from employment equals the expected discounted present value of his wage income plus the value to him of unemployment in the event the match dissolves (which occurs with flow probability δ). Notice that the integral in (2.9) captures the evolution of the worker's type q, over the course of the match. This is crucial, since in the event that the match subsequently dissolves the worker must re-enter the labor market with a suitably updated type, q, relevant for his new labor market experience. This consideration also explains the form of (2.10). If $\gamma_E = \gamma_G$ the worker's type q is invariant to the length of tenure in the current match, since the relative gain in human capital through on-the-job learning is exactly offset via the relative loss that occurs through general knowledge growth [see Eq. (2.5)]. The asset value (2.11) is interpreted in a similar manner. From the firm's perspective, the value of filling the vacancy, $\Pi(q)_F$, with a type q worker equals expected discounted revenues minus expected discounted costs. Finally, the value of holding open a vacancy, Π_V, equals the value of sampling a worker at random from the pool of unemployed workers ($\hat{\Pi}_F$) suitably discounted by the probability of finding a worker (η) and by the *growth* in average worker productivity (γ_G) over the sampling period.

2.6. Bargaining

A match between a searching worker with human capital $H(t) = H(t, s, q(\tau_E, \tau_U))$ and a vacancy may lead to a positive surplus that can be divided between the two parties through a suitable wage payment. Negotiations are conducted instantaneously according to a symmetric Nash bargaining solution.[11] Of course, even with an equal-division rule, the returns to each party are endogenous and depend upon both matching rates as well as the other variables of the model. The surplus accruing to type q worker-firm match is, $\max\{(J(q)_E + \Pi(q)_F - (J(q)_U + \Pi_V)), 0\}$. In a symmetric Nash bargain, this is equally divided between the bargaining parties, which in effective labor units gives,

$$J(q)_E - J(q)_U = \Pi(q)_F - \Pi_V \geqslant 0, \tag{2.14}$$

completing the formal description of the model.

[10] Although we impose this as an assumption, it may be derived as the outcome of a (continuous time) bargaining game in which (over a given epoch) the threat point, in the event of disagreement, is no production (see Binmore et al. [6]).

[11] With risk-neutral agents, the use of the Nash solution to bargaining problems in a dynamic environment is justified in Coles and Wright [9] via an application of sequential bargaining theory.

3. Steady states

In this section we determine: (i) the set of active job seekers, by evaluating the limit of integration Q, given in Proposition 1, (ii) the distribution of q in the populations U and E, (iii) the steady-state matching rate between unemployed workers and vacancies and (iv) the steady-state populations of job seekers and employed workers. We begin by deriving Q.

3.1. The terminal experience-adjusted vintage: Q

Lemma 1 (The terminal value Q). *The terminal employment-experience-adjusted vintage is given by*

$$Q = \ln[a(s, K_0)/\Pi_V] > 0 \tag{3.1}$$

which possesses the following properties:

(i) *Workers search for primary sector employment if and only if* $q \leqslant Q$.
(ii) (*Comparative statics*) $\partial Q/\partial s > 0$, $\partial Q/\partial K_0 > 0$, $\partial Q/\partial \Pi_V < 0$.

The inequality reported in (3.1) follows directly from the individual rationality condition embodied in the Nash bargaining solution (2.14). Part (i) of the Lemma indicates that if $q > Q$, the opportunity cost of filling the vacancy exceeds the potential benefit from the match, even if the worker accepts employment at a zero wage.[12] This finding is analogous to Akerlof's [5] notion of jobs as 'dam-sites'. However, there unemployment is generated by assuming that the number of unemployed individuals exceeds the exogenous supply of vacancies, whereas in this paper the population of vacancies is endogenously determined as part of the equilibrium. Part (ii) indicates that the terminal value Q is increasing in s and K_0. The reason is that these changes raise, for *each* q, the value of a worker-firm match. The boundary condition Q, completes the description of the asset-value equations reported in Proposition 1.

3.2. The distribution functions: $F(q)_U$ and $F(q)_E$

The primary-labor market is subject to the continual inflow β of new entrant workers as well as the outflow of older workers who either die, or retire to the secondary sector after reaching the terminal point Q described in Lemma 1. Furthermore, during the time that they are active in the primary sector, workers make a series of job to unemployment transitions and vice versa. This behavior induces a distribution of, q, in the extant populations of unemployed job seekers, U, and currently employed workers, E. Lemma 2 presents the associated steady-state cumulative probability-distribution functions, $F(q)_U$ and $F(q)_E$.

[12] If the fixed entry cost is completely replaced by a flow cost, the terminal value Q will be infinite. Yet, the new blood effect in forms of intergenerational rivalry will continue to exist, as seen in Proposition 2.

Lemma 2 (Distribution functions). (i) (*Distributional supports*) *If* $\gamma_E \leqslant \gamma_G$, *then* $q \in [0, Q]$, *otherwise* $q \in (-\infty, Q]$.

(ii) (*Cumulative distributions*) *The steady-state cumulative distribution functions,* $F(q)_U$ *and* $F(q)_E$ *are given by*

$$F(q)_U = G(q)_U / G(Q)_U \quad \text{and} \quad F(q)_E = G(q)_E / G(Q)_E, \tag{3.2}$$

where $G(q)_U$ *and* $G(q)_E$ *are the cumulative distribution functions of type q workers in, respectively, the populations of unemployed job seekers,* U, *and the currently employed,* E.

(iii) (*Population density functions*). *The respective population densities,* $g(q)_U$ *and* $g(q)_E$ *are given by*

$$g(q)_E = A_1 \exp[\lambda_1 q] + A_2 \exp[\lambda_2 q], \tag{3.3}$$

$$g(q)_U = A_3 \exp[\lambda_1 q] + A_4 \exp[\lambda_2 q], \tag{3.4}$$

where

$$A_1 = \frac{\mu\beta}{(\gamma_G - \gamma_E)(\lambda_1 - \lambda_2)} \quad \text{and} \quad A_2 = -A_1, \tag{3.5}$$

$$A_3 = \frac{\beta\{r_1(\gamma_G - \gamma_E) + \beta + \delta\}}{(\gamma_G - \gamma_E)(\lambda_1 - \lambda_2)}, \quad \text{and} \quad A_4 = -\frac{\beta\{r_2(\gamma_G - \gamma_E) + \beta + \delta\}}{(\gamma_G - \gamma_E)(\lambda_1 - \lambda_2)}, \tag{3.6}$$

and, λ_1 *and* λ_2 *are the* (*distinct and real*) *roots of the characteristic equation*:

$$\lambda^2 + \lambda\left[\frac{\beta + \delta}{\gamma_G - \gamma_E} + \frac{\beta + \mu}{\gamma_G + \gamma_U}\right] + \frac{(\beta + \delta)(\beta + \mu) - \delta\mu}{(\gamma_G - \gamma_E)(\gamma_G + \gamma_U)} = 0. \tag{3.7}$$

If $\gamma_E \leqslant \gamma_G$, then the definition of q in (2.5) implies that $q \geqslant 0$ (and that $q = 0$ if and only if $\tau_E = \tau_U = 0$). However, if $\gamma_E > \gamma_G$, then (2.5) indicates that q is decreasing in τ_E, giving rise to the support $(-\infty, Q]$ reported in part (i). If $\gamma_E = \gamma_U = 0$, the distribution functions take rather transparent forms as follows:

Corollary 1 (Distribution functions under $\gamma_E = \gamma_U = 0$). *The support of each distribution is* $[0, Q]$. *The cumulative population distribution functions, for each point q on this support are,*

$$\begin{aligned} G(q)_U = & \frac{\gamma_G}{\delta + \mu}\left\{\delta\left(1 - \exp\left[-\frac{\beta}{\gamma_G}q\right]\right)\right. \\ & \left. + \frac{\beta\mu}{\beta + \mu + \delta}\left(1 - \exp\left[-\frac{\beta + \delta + \mu}{\gamma_G}q\right]\right)\right\}, \end{aligned} \tag{3.8}$$

$$\begin{aligned} G(q)_E = & \frac{\gamma_G}{\delta + \mu}\left\{\mu\left(1 - \exp\left[-\frac{\beta}{\gamma_G}q\right]\right)\right. \\ & \left. - \frac{\beta\mu}{\beta + \mu + \delta}\left(1 - \exp\left[-\frac{\beta + \delta + \mu}{\gamma_G}q\right]\right)\right\}. \end{aligned} \tag{3.9}$$

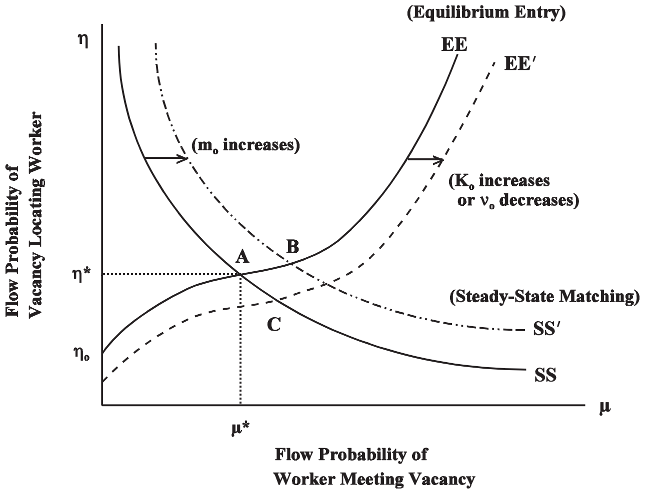

Fig. 1. Steady-state equilibrium and comparative statics.

3.3. Steady-state matching

The matching process between unemployed workers, U, and searching vacancies, V, is governed by Assumption 3. The constant-returns-to-scale property of $M(\cdot)$ in conjunction with (2.8) imply that the flow matching rate is given by,[13]

$$\mu U = \eta V = m_0 VM(U/V, 1). \tag{3.10}$$

Eq. (3.10) implies: $\eta = m_0 M(\eta/\mu, 1)$. Thus, it implicitly defines a function: $\eta = \eta^{SS}(\eta; m_0)$, which we refer to as the SS locus (see Fig. 1). The SS locus is the direct analogue of the Beveridge curve, drawn in matching parameter space μ–η, rather than in U–V space. Its properties, summarized in Lemma 3 below, follow immediately from the properties of the matching technology:

Lemma 3 (The *SS* locus). *Under Assumption* 3, *the function* $\eta = \eta^{SS}(\mu; m_0)$ *possesses the following limits*, $\lim_{\mu\to 0} \partial\eta^{SS}/\partial\mu = -\infty$ *and* $\lim_{\mu\to\infty} \partial\eta^{SS}/\partial\mu = 0$ *and possesses the properties*, $\partial\eta^{SS}/\partial\mu < 0$ *and* $\partial\eta^{SS}/\partial m_0 > 0$.

3.4. Steady-state populations

Finally, we note that the populations of unemployed and employed workers are governed by

$$\dot{U} = \{\beta + \delta E\} - \{g(Q)_{\mathrm{U}} + (\beta + \mu)U\}, \tag{3.11}$$

$$\dot{E} = \mu U - \{(\beta + \delta)E\}. \tag{3.12}$$

[13] Our results hold for any increasing quasi-concave function $M(U, V)$.

In steady-state the populations U and E are constant through time. Inspection of (3.11) indicates that the unemployment pool is constant if and only if the inflow into U, from new births (β) and job break-ups (δE), equals the outflow from successful search (μU), from deaths (βU), and from retirements ($(g(Q)_{\mathrm{U}})$. Similar considerations apply to (3.12).

4. The equilibrium wage

In this section, we characterize the equilibrium wage agreement, including the dispersion of wages in the population. We take as given the level of education $s \geqslant 0$ and the worker contact rate μ (these are subsequently endogenized in Sections 5 and 6). Moreover, to focus on essentials (namely economic growth and intergenerational rivalry), we ignore, for the moment, on-the-job-learning and the loss of skills while unemployed: $\gamma_{\mathrm{E}} = \gamma_{\mathrm{U}} = 0$.

4.1. The wage offer function

In this section, we examine the properties of the wage agreement between workers and firms. The terminal vintage Q, determined in Lemma 1, completes the formulation of the asset value functions given in Proposition 1. This enables us to derive the wage agreed under the Nash bargain (2.14). Consider:

Proposition 2 (The wage agreement). *There wage agreement, $w(q) = w(q; s, \mu, \gamma_{\mathrm{G}}, \Pi_{\mathrm{V}})$, determined in the symmetric Nash bargain between workers of $q \in [0, Q]$ and vacancies, is given by*

$$w(q) = \frac{\beta' + \mu}{\beta' + \frac{1}{2}\mu} \frac{a_0}{2} \exp[-q] - \frac{\beta' + \mu - \gamma_{\mathrm{G}}}{\beta' + \frac{1}{2}\mu - \gamma_{\mathrm{G}}} \frac{\Pi_{\mathrm{V}}}{2} + \frac{\frac{1}{2}\mu\gamma_{\mathrm{G}}}{(\beta' + \frac{1}{2}\mu)(\beta' + \frac{1}{2}\mu - \gamma_{\mathrm{G}})} \frac{\Pi_{\mathrm{V}}}{2} \exp\left[-\frac{(\beta' + \frac{1}{2}\mu - \gamma_{\mathrm{G}})}{\gamma_{\mathrm{G}}} \Delta Q\right], \tag{4.1}$$

where $\beta' \equiv \beta + r + \delta$ and $\Delta Q \equiv Q - q$. The wage agreement satisfies the following properties:

(i) $(1/w)\partial w/\partial q < -1$, *and* $w(Q) = 0$;
(ii) $\partial w/\partial s > 0$, $\partial w/\partial \mu > 0$, $\partial w/\partial \gamma_{\mathrm{G}} < 0$, *and* $\partial w/\partial \Pi_{\mathrm{V}} < 0$;
(iii) *For the workers of a given cohort* t^{b}, $\partial\{\exp[q]w(q)\}/\partial \gamma_{\mathrm{G}} < 0$ *for* $q < Q$.

In essence, the first two terms in (4.1) might be thought of as representing the outcome of a static Nash bargain between firms and a set of workers of *uniform* productivity $a_0 \exp[-q]$ (see Corollary 2 below), given a finite terminal vintage Q. Here, the term: $[\beta' + \mu - \gamma_{\mathrm{G}}]/[\beta' + (1/2)\mu - \gamma_{\mathrm{G}}]$ captures the option value to each firm of keeping the vacancy open and searching for another worker. The appearance of the growth rate, γ_{G}, reflects the value to the firm of waiting for more productive workers in the future. The third term represents the *capital gain* that workers experience as q increases. It reflects the ability of workers of older generations (higher q) to capture some of the benefits accruing to firms from the economic growth embodied in *new entrant* workers.

The proportionate rate of decline of expected discounted revenues, $a_0 \exp[-q]$, with respect to q, is simply unity. However, as part (i) of the Proposition indicates, the wage declines at an *even more* rapid pace due to the existence of a terminal vintage as a result of intergenerational competition. Crucially, the closer a worker is to Q the greater the cost to him of disagreement, since in this event he must search afresh and generate a new job offer before reaching Q. As a result workers' bargaining positions atrophy as q rises, which provides the additional impetus for the proportionate reduction of the wage with q. Part (i) indicates also that once the terminal point Q is reached the wage equals zero, which is the normalized wage available in the competitive-secondary sector. Once workers reach this point, they cannot secure employment at a positive wage, despite the fact that they possess a positive *marginal productivity*, $a(s, K_0)\exp[-Q]$. As in Akerlof [5], this feature might be thought of as corresponding to a form of involuntary unemployment.

Part (ii) of the Proposition summarizes the more interesting comparative static properties of the wage agreement. An increase in education, s, raises the worker's human capital which raises the present value of the revenues accruing from the match: $a(s, K_0)\exp[-q]$. This augments, regardless of q, the size of the 'pie' that must be divided between the two parties (without altering either side's relative bargaining position), so that the wage w rises accordingly. A faster matching rate of workers with vacancies, μ, makes *alternative* options more accessible to workers, which raises the wage. An increase in γ_G raises the growth rate of public knowledge [see Eq. (2.1)], which stiffens the extent of intergenerational competition among workers and lowers w accordingly. Finally, an increase in Π_V, raises the value of the firm's outside opportunity, which reduces the wage. Part (iii) considers the effect of an increase in γ_G upon a worker's *real* human wealth as opposed to his wealth measured in effective labor units. The date t real wealth of a worker with experience-adjusted vintage, $q \equiv \tau\gamma_G$, is found by multiplying the wage $w(q)$ by the effective labor unit factor $\exp[\gamma_G t] \equiv \exp[q + \gamma_G t^b]$, to yield: $\exp[t^b\gamma_G]\{\exp[q]w(q)\}$. A marginal increase in γ_G, *at date* t^b, pushes up the knowledge frontier, $K(t)$ at each *subsequent* date. The effect of this for the cohort born at t^b, is to worsen severity of intergenerational competition at each point in their future. The anticipated effect of this is fully embodied in the asset value equations, leading to the reported reduction in real incomes. The notion that older searchers may be disadvantageous relative to younger ones has some support in the empirical literature. For example, Fallick [14, p. 17] notes that: "experienced workers may be expected to have greater difficulty in moving into expanding sectors of the economy than do similarly situated new entrants." Furthermore, Neuman and Weiss [28] find that obsolescence of human capital plays a significant role in explaining differences between wage profiles in high and low-tech oriented industries.

To see the crucial role played by the assumption that the community's stock of knowledge continually grows through time, consider Corollary 2:

Corollary 2 (The wage offer in the absence of a new blood effect). *In the limit*, $\gamma_G \to 0$, *the wage* $w(q)$ *converges a.e. to*,

$$w(0) = \frac{\beta' + \mu}{\beta' + \frac{1}{2}\mu} \frac{\{a_0 - \Pi_V\}}{2}. \tag{4.2}$$

In the absence of the continual accumulation of public knowledge, $\gamma_G \to 0$, the cutting-edge of intergenerational rivalry is removed. As a consequence workers of each generation are identical *a.e.* and search until they either locate employment or else exit from the primary labor market.[14] Indeed, in this case, the wage agreement converges, in essence, to that derived by Diamond [13].

4.2. *The distribution of wages*

We next turn to examining the steady-state distribution of wage across workers with different job histories and extend the model to look at wage patterns across workers with differing innate abilities.

4.2.1. *Wage disparity among workers with differing job histories*

Our model generates an endogenous steady-state distribution of wage income across workers with differing histories and hence types, q. In this subsection we focus on the wage distribution conditioning upon a common employment date t' (for example, think of the today's distribution of wages across all workers who secured their current job in 1990).[15] Wages in this cross-section vary according to the (random) time that workers spent unemployed prior to securing employment, reflecting the effects of intergenerational rivalry. Ignoring job break-ups (i.e., $\delta = 0$), the density of employed workers who secured employment after searching for a period $\tau \equiv \tau_U$ is

$$g(\tau)_E = \frac{(\mu + \beta)\exp[-(\mu + \beta)\tau]}{1 - \exp[-(\mu + \beta)T]} \quad \text{for } \tau \in [0, T], \tag{4.3}$$

where $T = Q/\gamma_G$ is the terminal search horizon. The density $g(\tau)_E$ reflects the notion that the distribution of workers who obtain employment at date t' is a biased sample of the population. In particular, the greater τ, the greater the probability that a worker either died previously or else obtained employment at a date *prior* to t'.

We focus on the variance of the wage distribution, which is the simplest measure of dispersion in the Kolm–Pollak class of absolute, decomposable, and symmetric

[14] More precisely, the distribution of population degenerates for all $q > 0$ and has a mass point for $q = 0$.

[15] We have also examined the wage distribution of a given cohort $t^b < t$—for example, think of the current distribution of wages of workers born in 1961, who have survived, and are currently employed. The results of this exercise are very similar to those presented here, and hence not reported.

indexes.[16] Using Proposition 2 and (4.3) we can calculate the variance of the distribution, $Var[w]$ (see the Appendix). Under zero profit ($\Pi_V = v_0$, to be discussed in Section 5), we have $Q = \ln[a_0/v_0]$ which depends on a_0 and v_0. For given μ, analytically, we can prove that $Var[w]$ is locally increasing (and concave) in γ_G when γ_G is sufficiently small and that it approaches zero as $\gamma_G \to 0$ or $\gamma_G \to \infty$. Thus, wage dispersion measure is hump-shaped with respect to the exogenous rate of knowledge growth and hence the severity of human capital obsolescence.

Next, to better characterize $Var[w]$, we perform simple numerical exercises, as further analytical derivations become quickly intractable. We select the benchmark values for the relevant parameters based primarily on the US experience. More specifically, we set (i) $\beta = 0.01$ (roughly in accord with the US population growth rate), (ii) $r = 0.035$ (iii) $\gamma_G = 0.01$ (this corresponds to the exogenous component of economic growth), (iv) $a_0 = 1.5v_0$ and normalizing $a_0 = 1$ (since normalizing a_0 to unity and assuming that total labor and entrepreneurial income is 1.5 times greater than capital income gives an acceptable steady-state terminal horizon, $T = 40$), (v) $\mu = 0.95$ (based on the steady-state population growth rate and the assumption that the natural rate of unemployment or the percentage of the secondary market activity is 5%).

The main results are as follows. First, in the benchmark case we find: $\arg\max_{\gamma_G} Var[w] = 0.0075$ this value drops to 0.0037 when $a_0/v_0 = 1.2$ and increases to 0.0102 when $a_0/v_0 = 1.8$. Thus, with a "moderate" rate of knowledge growth, say between 0.4% and 1%, the variance of wages across different birth cohorts is high, while with extremely low or high rates of knowledge growth the variance of wages becomes low (see the solid curve in Fig. 2, which depicts the benchmark case). This may help explaining why in the 1940s (following the Great Depression when γ_G was low) the US experienced a wage compression across cohorts, while in the 1960s (following a period of prolonged and smooth growth) it experienced a wage expansion (increased dispersion). Second, for a broad range of parameter values, the variance of wages across different birth cohorts is increasing in a_0 and decreasing in v_0. Numerically, we find that this is due mainly to the additional advantage of the young over the old, which results from a longer terminal vintage.

Finally, when γ_G is sufficiently small and β is not too small, our numerical results suggest that an increase in the worker's contact rate (μ) tends to decrease the variance. Intuitively, this is because a higher μ reduces the severity of intergenerational competition (i.e., workers quickly secure employment and they need not fear obsolescence), thereby leading to a compression of the wage distribution. This finding can explain, at least partially, the US wage expansion in the 1980s. Specifically, the 1979–82 second oil crisis and the subsequent increase in the number of two-worker households caused a sharp increase in the US unemployment rate and a decrease in the contact rate with which workers located employment in the primary labor market. The latter led, in turn, to an increase in the variance of the wage distribution (wage expansion).

[16] The reader is referred to Kolm [18] and Pollak [33] for detailed definitions and illustrations.

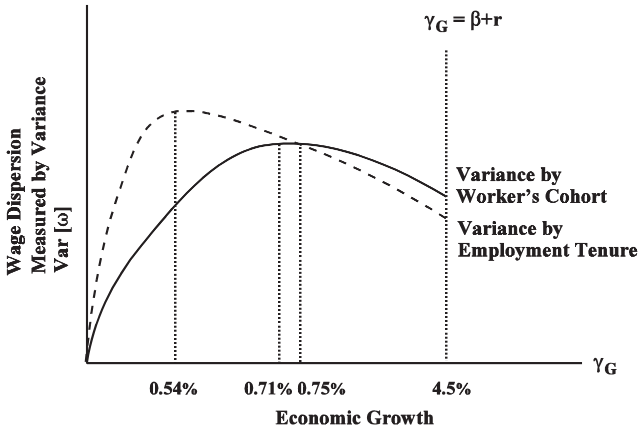

Fig. 2. Wage dispersion versus income growth.

These predictions of the model give theoretical support to the empirical findings of Goldin and Margo [15], Murphy and Welch [27] and Solon et al. [36], regarding the wage compression and wage expansion experienced in the US after the Great Depression. The primary reason that our framework is capable of resolving the wage compression/expansion puzzle is due to the interaction between economic growth and intergenerational rivalry, which results in a non-monotonic relationship between wage dispersion and income growth.

4.2.2. *Heterogeneous worker ability*

The empirical wage-dispersion literature has also uncovered some important differences between the wages earned by high- and low-ability workers (see, for example, Neuman and Weiss [28]). Our model is readily extended to admit differences in worker ability. Assume that the level of education, s, is given. Letting H connote "high" and L "low", we consider the simplest case in which worker productivity takes one of two values, $a_H > a_L > v_0$ [see Eq. (2.12)]. To avoid issues of asymmetric information, we assume that $a \in \{a_L, a_H\}$ is known to both the worker and the firm at the time of employment.[17] Using (3.1), it is easily seen under zero profit that the terminal values Q_H and Q_L now satisfy,

$$\Delta Q(a) \equiv Q_H - Q_L = \ln(a_H/a_L). \tag{4.4}$$

Using (4.1) and assuming a predetermined value of the worker-firm contact rate μ (this is the case if the SS locus is near vertical), the wage ratio of high- to low-skilled

[17] An interesting extension of the present work is to assume that $a \in \{a_L, a_H\}$ is private information. In this case, the time the worker spends unemployed could provide important information about the worker's ability. A dynamic adverse selection model in which time on the market acts as a signal of ability is examined by Taylor [38].

workers is,

$$\frac{w_H(q)}{w_L(q)} = \frac{a_H A_5 \exp[-q] + \gamma_G(\frac{1}{2}\mu)\exp\left[-\frac{(\beta' + \frac{1}{2}\mu - \gamma_G)}{\gamma_G}\Delta Q_H\right]v_0 - A_6 v_0}{a_L A_5 \exp[-q] + \gamma_G(\frac{1}{2}\mu)\exp\left[-\frac{(\beta' + \frac{1}{2}\mu - \gamma_G)}{\gamma_G}\Delta Q_L\right]v_0 - A_6 v_0}, \tag{4.5}$$

where $A_5 \equiv (\beta' + \mu)(\beta' + \frac{1}{2}\mu - \gamma_G)$ and $A_6 \equiv (\beta' + \frac{1}{2}\mu)(\beta' + \mu - \gamma_G)$. Straightforward manipulation and differentiation yield,

$$\lim_{\gamma_G \to 0} \frac{w_H(q)}{w_L(q)} = \frac{a_H - v_0}{a_L - v_0} > \frac{a_H}{a_L}, \tag{4.6}$$

$$\lim_{\gamma_G \to 0} \frac{\partial(\frac{w_H(q)}{w_L(q)})}{\partial \gamma_G} \propto \left[q + \frac{\mu}{(\mu + \beta')(\frac{1}{2}\mu + \beta')}\right]\frac{v_0}{2}(a_H - a_L) > 0. \tag{4.7}$$

Notice from (4.6) that, for small values of γ_G, the wage differential exceeds the underlying skill differential. Furthermore, (4.7) indicates that the magnitude of the differential increases with the extent of intergenerational rivalry (γ_G). The reason is that a greater initial ability, $a \in \{a_L, a_H\}$ buys workers additional time before their human capital is rendered obsolete. This search advantage enhances the bargaining power of high-skill workers, which increases their relative wage over and above that which is warranted on the basis of their skill differential alone.

Finally, consider the wages and wage differentials as search frictions and the fixed entry cost, v_0, become vanishingly small:

$$\lim_{v_0 \to 0}\left\{\lim_{\mu \to \infty} w(q \mid a)\right\} = a\exp[-q],$$

$$\lim_{v_0 \to 0}\left\{\lim_{\mu \to \infty} [w(q \mid a_H) - w(q \mid a_L)]\right\} = (a_H - a_L)\exp[-q].$$

Thus, in the absence of market (search and entry) frictions, wages converge to their 'competitive' values (wherein workers are paid a wage equal to their expected discounted marginal products). The importance of this result is that it implies market frictions, whether through finite μ or through positive v_0 are crucial for the effects of intergenerational rivalry to become manifest.

5. General equilibrium search

In this section, we derive the general equilibrium properties of the model, by endogenizing the flow contact rates μ^* and η^*, which up to this point have been treated as exogenous. Once these values are determined, we exploit the model's recursive structure to determine the values of the other endogenous variables. The key construct that enables us to accomplish this goal in a tractable manner is the equilibrium-entry (*EE*) locus, which gives the set of (μ, η) combinations for which firms earn zero ex ante profits. Crucially, the *EE* locus embodies the behavior of the equilibrium wage outlined in Proposition 2. Steady-state equilibrium values of the contact rates (η, μ); are given as point(s) of intersection of this locus with the steady-state (*SS*) relationship described in Lemma 3. We begin with the following definition:

Definition. A steady-state equilibrium consists of a terminal vintage Q^*, distribution functions $F^*(q)_{\mathrm{U}}$, $F^*(q)_{\mathrm{E}}$, a wage agreement $w^*(q)$, and a quadruple $(\mu^*, \eta^*, U^*, V^*)$ that satisfies Lemmas 1–3, Proposition 2, equations $\dot{E} = 0$ and $\dot{U} = 0$ [see (3.12) and (3.11)], and the equilibrium-entry Condition,

$$\Pi^*_{\mathrm{V}} = v_0. \tag{5.1}$$

The first part of the definition ensures that the functions, Q^*, $F^*(q)_{\mathrm{U}}$, $F^*(q)_{\mathrm{E}}$, and $w^*(q)$ are consistent with the partial-equilibrium relationships derived earlier. Together, Lemma 3, equations $\dot{E} = 0$ and $\dot{U} = 0$ and (5.1) ensure stationary populations in the steady state.

5.1. Equilibrium entry of firms

Using (2.13), the free-entry condition (5.1) can be written as

$$\Pi^*_{\mathrm{V}} \equiv [\eta/(\eta + \delta - \gamma_{\mathrm{G}})]\hat{\Pi}^*_{\mathrm{F}} = v_0, \tag{5.2}$$

where $\hat{\Pi}^*_{\mathrm{F}} \equiv \int_0^Q \Pi(q)_{\mathrm{F}}\, dF(q)_{\mathrm{U}}$ is the expected value to the firm of drawing a worker at random from the pool of unemployed job seekers. Eq. (5.2) implicitly defines the equilibrium entry (*EE*) locus: $\eta = \eta^{\mathrm{EE}}(\mu; a_0; \gamma_{\mathrm{G}}, v_0)$, which, for each μ, gives the value of η at which firms just break even upon entering the primary sector (see Fig. 1).

Lemma 4 (The *EE* locus). *The function* $\eta = \eta^{EE}(\mu; a_0, \gamma_{\mathrm{G}}, v_0)$ *possesses the following properties*:

(i) $\lim_{\mu \to 0} \eta^{EE}(\mu, a(s(\mu), \cdot); \cdot) = \eta_0 \in (0, \bar{\eta})$ *with* $\bar{\eta} < \infty$, *and* $\lim_{\mu \to \infty} \eta^{EE}(\mu, a(s(\mu; \cdot), \cdot) = \infty$;

(ii) *for all* $s \geqslant 0$, $\partial\eta^{EE}/\partial a_0 < 0$, $\lim_{\gamma_{\mathrm{G}} \to 0} \partial\eta^{EE}/\partial\mu > 0$, *and* $\lim_{\gamma_{\mathrm{G}} \to 0} \partial\eta^{EE}/\partial\gamma_{\mathrm{G}} > 0$, *if* $\mu \in (0, \bar{\mu})$, $\bar{\mu} < \infty$, *and* $\lim_{\gamma_{\mathrm{G}} \to 0} \partial\eta^{EE}/\partial\gamma_{\mathrm{G}} < 0$ *otherwise.*

The *EE* locus is ‘well behaved,’ being continuous, starting at a positive value of $\eta = \eta_0$ and approaching infinity as μ approaches infinity (part (i)). Part (ii) outlines the basic properties of the *EE* locus. An increase in $a_0 \equiv a(s, K_0)$ raises the profitability of hiring a worker of each type, q. This, in turn, increases the profits accruing to each match, $\Pi(q)_{\mathrm{F}}$, which stimulates the entry of vacancies and tends to lower η. However, the increase in a_0 also, by raising $Q = \ln[a_0/v_0]$, spreads out the distribution $F(q)_{\mathrm{U}}$ which for given $\Pi(q)_{\mathrm{F}}$, tends to lower $\hat{\Pi}_{\mathrm{F}}$ and to raise η. We prove that the former of these effects dominates, so that η is increasing in a_0. Given that, $\partial a_0/\partial s > 0$, an immediate corollary of this finding is that, $\partial \eta^{EE}/\partial s < 0$. The effect of an increase in the worker contact rate, μ, is in general ambiguous. On the one hand, as indicated in Proposition 2, an increase in μ raises the wage, which lowers expected profits $\hat{\Pi}^*_{\mathrm{F}}$ and deters entry (which raises η). On the other hand, there is a favorable effect upon the distribution $F(q)_{\mathrm{U}}$. The reason is that an increase in μ decreases the average duration of search and lowers the probability that workers remain unmatched before reaching the terminal point Q. This tends to *raise* the productivity of the *average* job seeker, which encourages entry and lowers η. Which of these two effects dominates depends upon the relative cost of a higher wage versus the benefit of facing a relatively more productive population of job seekers. If γ_{G} is small enough the wage effect dominates, since workers are nearly homogeneous anyway and there is therefore little benefit from any improvement in the distribution. As indicated in part (ii) of the Lemma, under these conditions, η is strictly increasing in μ. An increase in the growth rate of knowledge, γ_{G}, also leads to two conflicting tendencies. First, it stiffens the severity of intergenerational competition and decreases the wage, w (Proposition 2). This stimulates the entry of new vacancies and lowers η. Second, it lowers the *average* productivity (in effective units) of job-searchers, which tends to raise η.[18] As indicated, either effect may dominate. However, for low values of μ the productivity effect dominates, since wage payments are low anyway (Proposition 2) and are relatively unresponsive to changes in γ_{G}. For large enough values, of μ, the contact rate η is increasing in γ_{G} for similar, but opposite, considerations.

5.2. *Steady-state equilibrium*

We now prove the existence of a steady-state search equilibrium and characterize its properties.

Proposition 3 (Steady-state equilibrium). *Under Assumptions* 1–3 *and Condition N*, *a non-degenerate steady-state equilibrium exists. For sufficiently small values of* γ_{G}, *the equilibrium is unique*.

Existence is trivial. Lemma 3 shows that the *SS* locus begins at infinity and approaches zero asymptotically as μ, while Lemma 5 shows that the *EE* locus begins at $\eta_0 > 0$ and that $\lim_{\mu\to\infty} \eta^{EE} = \infty$. Since both functions are continuous, there must exist at least one equilibrium (point A in Fig. 1), where the *EE* locus cuts the *SS* locus from below. The equilibrium is unique if the *EE* locus is strictly increasing in μ, which is the case for sufficiently small values of the growth rate, γ_{G} (Lemma 4). We summarize the most interesting comparative-static properties of the equilibrium below.

[18] Recall that costs increase at the same rate as firm's expected profits, γ_{G}. Therefore, only benefits/costs of effective units are relevant at the margin.

Proposition 4 (Properties of steady-state equilibrium). *For any given* $s \geqslant 0$ *and for small* $\gamma_G > 0$, *we have*:

(i) (*wages*) $dw^*/ds > 0$, $dw^*/dK_0 > 0$, $dw^*/dm_0 > 0$;
(ii) (*unemployment*) $dU^*/ds < 0$, $dU^*/dK_0 < 0$, $dU^*/dm_0 < 0$.

These results are intuitive. For instance, an increase in a_0—induced by an increase in s or in K_0—shifts down the EE locus to EE' along the given SS locus (see Fig. 1). This reduces the equilibrium worker contact rate μ^* which: (i) works in tandem with the reduction in a_0 to lower the wage Proposition 2) and (ii) increases the unemployment rate, by making it more time consuming for workers to find jobs. A reduction in matching frictions, which increases m_0, shifts out the SS locus to SS' along a given EE schedule (see Fig. 1). This unambiguously increases μ^*, which raises w^* (Proposition 2) and lowers the level of unemployment U^*.

6. Education and re-training

In this section, we endogenize the human-capital investment decision, including both basic education and worker retraining.

6.1. Educational choice

We begin by determining the properties of the optimal schooling decision, and then offer a brief discussion of its efficiency properties.

6.1.1. Equilibrium schooling

Greater investment in schooling is (privately) beneficial to workers, in that it raises their human capital upon entry into the labor market and consequently their wage income once employed. Nevertheless, education is also costly due to either a direct pecuniary cost or the disutility associated with effort. Let $c(s)$ denote the utility cost to workers (per effective labor unit) of effort s.[19] We assume that $c(s)$ is strictly increasing and strictly convex in s and that it satisfies $dc(0)/ds = 0$. Each worker, treating the contact rates μ and η as parametric, chooses s to solve program (P): $\max_{s \geqslant 0}\{J(0)_U - c(s)\}$, where $J(0)_U$ is the worker's value upon entering the labor market (at which point $q = 0$). Assumption 1 implies that for small $\gamma_G \geqslant 0$, the program (P) possesses a unique interior maximum. The first-order

[19] Our analysis is easily emended, without altering the main results, to the case in which schooling is time consuming by considering the program: $\max_s \exp[-(\beta + r)s]J(0)_U - c(s)$, where s is the time spent on education. (It might be observed that the asset value function $J(0)_U$ is suitably discounted by the length of schooling time, s.) The present approach is adopted since it is somewhat simpler.

condition is

$$\partial J(0)_{\mathrm{U}}/\partial s = c'(s). \tag{6.1}$$

Consider,

Condition U. *For all* $s \geqslant 0$, $\frac{[\partial a_0/\partial s]^2}{\beta[a_0 - v_0]} < c''$.

We can then characterize the partial equilibrium behavior of worker's educational choices:

Lemma 5 (Schooling effort). *The individual schooling effort function,* $s = s(\mu, \gamma_{\mathrm{G}}, K_0)$ *satisfies*: $\partial s/\partial\mu > 0$, $\partial s/\partial\gamma_{\mathrm{G}} < 0$ *and* $\partial s/\partial K_0 > 0$.

Factors which influence the wage or matching probabilities alter the incentives to invest in education, and therefore the amount undertaken by workers. Thus, under Condition U, Proposition 2 implies an increase in the contact rate μ raises s, by increasing the marginal returns to education, while an increase in γ_{G} lowers s, by increasing the severity of intergenerational competition.

The properties of the steady-state equilibrium allowing for endogenous adjustments in education, s, are broadly similar to those reported in Propositions 3 and 4, with one important caveat. Recall that the *EE* locus reported in Lemma 4 is a function: $\eta = \eta^{EE}(\mu; a_0; .)$, where $a_0 \equiv a(s, K_0)$. Consequently, as explored in Laing et al. [19] once s is recognized as endogenous, the *EE* locus is seen to depend both directly upon the contact rate μ and indirectly upon it through s and hence a_0. While the *EE* locus is increasing in μ holding s constant, (at least for small enough values of γ_{G}), this may not be the case once s adjusts. The reason is that higher values of μ raises both the wage, w^* *and* the level of schooling, s^*. The former of these effects discourages entry (η rises), while the latter effect encourages it (η falls). This implies multiple equilibria may arise in our model, but for the purposes of this paper we focus on examining the properties of an equilibrium satisfying Samuelson's Correspondence Principle (at which the *EE* locus is upward-sloping):[20]

Proposition 5 (Steady-state equilibrium with endogenous schooling). *Under Assumptions* 1–3 *and Condition N, a non-degenerate steady-state equilibrium exists. Under Condition U and for small enough* γ_{G}, *the equilibrium possesses the following properties*: *there exists an* $\tilde{m}_0$ *such that, for all* $m_0 \in (0, \tilde{m}_0)$, $\partial w^*/\partial\gamma_{\mathrm{G}} < 0$, $\partial s^*/\partial\gamma_{\mathrm{G}} < 0$, *and* $\partial U^*/\partial\gamma_{\mathrm{G}} > 0$.

Lemma 4 showed that an increase in γ_{G} causes the *EE* locus to pivot around a given contact rate μ. For small enough values of $m_0 \leqslant \tilde{m}_0$ this effect must lead to a reduction in the equilibrium contact rate μ^*. Under these circumstances, the increase

[20] Although of interest, the implications of multiple equilibria are not discussed here as they are similar to those analyzed in Laing et al. [19].

in γ_G and reduction in μ work together to unambiguously lower the equilibrium wage, reduce the level of schooling and raise the equilibrium level of unemployment. Once again, these results reflect the effects of intergenerational competition. In particular, an increase in the growth rate of knowledge leads to the more rapid obsolescence of workers' human capital, which discourages workers' educational efforts. In turn this reduces the wage and discourages the entry by firms (which then induces the increase in U^*).

6.1.2. Welfare

In order to enrich the welfare analysis it is of interest to admit interactions between the rate of growth, γ_G (and hence the extent of intergenerational rivalry) and the level of worker education, s. To do so, we assume that γ_G, depends positively on the *average* level of workers' education denoted, $\bar{s}$, reflecting uncompensated knowledge spillovers as in Romer [34] and Lucas [22]. More precisely, let $\gamma_G = \Gamma_G(\bar{s})$, where Γ_G is strictly increasing, strictly concave, and is bounded by r (so that present discounted values are bounded). A steady-state equilibrium with *endogenous growth* is derived as a Nash equilibrium in which each worker's optimal choice of education coincides with some given average level of educational attainment in the workforce. Around any such point where $s = \bar{s}$, an increase in the level of education raises both individual productivity $a(s, K_0)$ and the rate of economic growth $\Gamma_G(\bar{s})$. Provided that, in the neighborhood of $s = \bar{s}$, the direct effect of s on the valuation $a(s, K_0)$ dominates the indirect growth effect (via $\bar{s}$), the comparative-static results reported in Propositions 4 and 5 remain unchanged and hence are not repeated.

We now consider the social optimality of workers' educational choices. We assume that the social planner's objective is the maximization of the steady-state value of: $J(0)_U - c(s)$, which is the expected discounted utility of a date $t = 0$ representative new entrant worker. We conceive of the social planner as respecting the economic environment (tastes and both production and matching technologies), but determining the level of education s^{so} chosen by workers. Crucially, the social planner recognizes that her choice of s^{so} affects entry by firms as well as the steady-state contact rate μ^{so}. The socially optimal schooling effort, s^{so}, must then satisfy:

$$\left[\frac{\partial J(0)_U}{\partial a_0} + \frac{\partial J(0)_U}{\partial Q}\frac{\partial Q}{\partial a_0}\right]\frac{\partial a_0}{\partial s} + \frac{\partial J(0)_U}{\partial \mu}\frac{\partial \mu^{so}}{\partial s} + \frac{\partial J(0)_U}{\partial \gamma_G}\frac{\partial \Gamma(s^{so})}{\partial s} = \frac{dc}{ds}. \tag{6.2}$$

In a decentralized equilibrium, each worker considers only the direct benefits of schooling, given by the first term on the left-hand side of (6.2). This includes the benefits accruing from a greater 'pie' to be divided between the bargaining parties (a_0) as well as from an enhanced bargaining position due to an increase in the terminal value of Q. The second term in (6.2) corresponds to the training externality first identified by Acemoglu [1]. More specifically, individuals ignore the beneficial impact of their effort on the subsequent profitability of firms. In particular, increases in average education levels induce entry by firms, raising the contact rate μ^{so} and hence workers' welfare. The third term in (6.2) is of particular interest in the context of the current paper, in that individuals fail to account for the deleterious effects of their educational efforts on the rate of economic growth, $\Gamma(s)$, and hence upon the rate at which the human capital of each generation is rendered obsolete. As a result of these considerations, it is difficult to be precise about the social efficiency of educational levels in the decentralized equilibrium, since the former externality indicates a tendency towards underinvestment whereas the latter points towards overinvestment.

6.2. Re-training

Up to this point we assume that each worker's relative productivity inexorably declines with his adjusted employment experience, q. We now consider a more proactive environment wherein costly re-training allows workers to recover any losses in human capital they may have suffered over the course of their career. This leads to an additional source of new-blood; the entry of newly re-trained workers. In order to focus on essentials, we assume that both the level of primary education, $s \geq 0$ and the worker contact rate, μ are given (which corresponds to a vertical SS curve Lemma 3). We assume that at a cost $C_0 > 0$, a worker can (instantly) re-train and acquire the most recent 'state-of-the-art' knowledge, $k(s,t)$ [Eq. (2.2)].[21]

Critical re-training points are defined by the triple: $(\hat{q}^{\mathrm{U}}, \hat{q}_0^{\mathrm{E}}(q), \hat{q}^{\mathrm{E}})$ where: $\hat{q}^{\mathrm{U}}$ is the point at which an unemployed worker (periodically) re-trains himself, $\hat{q}_0^{\mathrm{E}}(q) \geq q$ is the point at which a *newly hired* type q worker is first re-trained. Finally, $\hat{q}^{\mathrm{E}}$ is the point at which an established (i.e., once re-trained) employee is periodically re-trained at a given firm. If the worker never re-trains himself while unemployed, then $\hat{q}^{\mathrm{U}} \to \infty$. Similar remarks apply to the limiting values of $(\hat{q}_0^{\mathrm{E}}(q), \hat{q}^{\mathrm{E}})$. We assume that $\hat{q}^{\mathrm{U}}$ is chosen by the worker to maximize his valuation, $J(q)_{\mathrm{U}}$ and that the pair $(\hat{q}_0^{\mathrm{E}}(q), \hat{q}^{\mathrm{E}})$ are jointly chosen by the worker and his employer to maximize the surplus from the match. The asset value functions $J(q)_{\mathrm{U}}$ and $J(q)_{\mathrm{E}}$ incorporating retraining are somewhat lengthy (see the appendix) and at this level of generality, the re-training decision is quite complicated. However, considerable progress can be made if we impose,

Condition R. (i) $\gamma_{\mathrm{E}} = \gamma_{\mathrm{G}} \geq 0$ (ii) $a_0 - C_0 - v_0 > 0$.

With $\gamma_{\mathrm{E}} = \gamma_{\mathrm{G}}$, the worker's type q is invariant to the length of tenure at a given job (since the relative loss in skill through intergenerational competition is exactly offset by the growth of skills on the job). Part (ii) is necessary (as we shall see) for any re-training to be undertaken in equilibrium.

Lemma 6 (Re-training on the job). *Under Condition R,* (i) *re-training occurs at most once on a given job ($\hat{q}^{\mathrm{E}} \to \infty$ and either $\hat{q}_0^{\mathrm{E}}(q) < \infty$ or $\hat{q}_0^{\mathrm{E}}(q) \to \infty$) and* (ii) *if re-training takes place it is done at the point of hiring ($\hat{q}_0^{\mathrm{E}}(q) = q$).*

[21] We thank two anonymous referees for proposing that we extend our model in this direction.

Given $\gamma_E = \gamma_G$, if the worker is re-trained, there is obviously no point of re-training him again, since he *is and remains* the newest blood. This leaves the initial re-training decision $\hat{q}_0^E(q)$. Given that the worker's state q is independent of the length of tenure at the job, if re-training is warranted then, with positive discounting, it must be optimal to do so immediately. Even under condition R(i), the problem is not trivial. The worker may re-train numerous times during a given unemployment spell and although re-trained at most once on a given job, the worker may have many jobs.

Consider a match between a given firm and a type q worker. We assume that on-the-job re-training is carried out if it is pairwise efficient, in the sense of maximizing the surplus from the match. We further *assume* (and later prove) that workers do not re-train themselves. The re-training decision is solved as a Nash equilibrium. Let the worker's alternative market options be characterized by the triple, $(Q_E', J(q)_E', J(q)_U')$ where Q_E' is the (common) point at which other firms in the market re-train the worker and $J(q)_E'$ and $J(q)_U'$ are the worker's market asset values (outside options). If the worker exercises his outside options, he anticipates re-training if and only if $q \geqslant Q_E'$. With the assumption that the re-training cost C_0 is independent of the worker's state q, all workers with $q \geqslant Q_E'$ are treated equally in the market. We first determine the re-training value Q_E at the given firm conditional upon the triple $(Q_E', J(q)_E', J(q)_U')$ available in the market. Let $S(q, I)$ represent the surplus generated from the given match, where $I = 1$ if the firm re-trains the worker and $I = 0$ otherwise. We have,

$$S(q,1) = (a_0 - C_0) + \hat{\delta}J(0)_U' - (\hat{\delta}J(q)_U' + v_0), \tag{6.3}$$

$$S(q,0) = a_0 \exp[-q] + \hat{\delta}J(q)_U' - (\hat{\delta}J(q)_U' + v_0), \tag{6.4}$$

where $\hat{\delta} \equiv \delta/(\tilde{\beta} + \delta)$ and $\tilde{\beta} \equiv \beta + r - \gamma_G$ Conditional upon the worker being hired, re-training is optimal from the perspective of the worker-firm pair if,

$$S(q,1) - S(q,0) = (a_0(1 - \exp[-q]) - C_0) + \hat{\delta}(J(0)_U' - J(q)_U') \geqslant 0. \tag{6.5}$$

A Nash equilibrium is defined by, $Q_E = Q_E' \equiv Q_E^* \geqslant 0$ such that,

$$(a_0(1 - \exp[-Q_E^*]) - C_0) + \hat{\delta}(J(0)_U - J(Q_E^*)_U) = 0. \tag{6.6}$$

Notice that, quite naturally, the re-training decision depends upon the suitably-discounted capital gain the worker enjoys if the match subsequently dissolves (which occurs with discounted-flow probability $\hat{\delta}$). Although we do not compute the social optimum, we note that, as in Acemoglu [1], the worker-firm pair fails to recognize the beneficial effects of the re-training decision upon the universe of alternative prospective future employers.

Conditional upon the value of $Q^*_{\rm E}$ the worker's asset value functions are given by

$$J(q)_{\rm E} = w(q) + \hat{\delta} J(q)_{\rm U}, \quad q < Q^*_{\rm E}, \tag{6.7}$$

$$J(q)_{\rm U} = \frac{\mu}{\gamma_{\rm G} + \gamma_{\rm U}} \int_q^{Q^*_{\rm E}} \exp\left[-\frac{\tilde{\beta} + \mu}{\gamma_{\rm G} + \gamma_{\rm U}} (q' - q) \right] J(q')_{\rm E}\, dq' + \hat{\mu} \exp\left[-\frac{\tilde{\beta} + \mu}{\gamma_{\rm G} + \gamma_{\rm U}} \Delta Q^*_{\rm E} \right] J(Q)_{\rm E}, \quad q < Q^*_{\rm E}, \tag{6.8}$$

$$J(Q^*_{\rm E})_{\rm E} = w(Q^*_{\rm E}) + \hat{\delta} J(0)_{\rm U}, \quad q \geqslant Q^*_{\rm E}, \tag{6.9}$$

$$J(Q^*_{\rm E})_{\rm U} = \hat{\mu} J(Q^*_{\rm E})_{\rm E}, \quad q \geqslant Q^*_{\rm E}, \tag{6.10}$$

where $\Delta Q^*_{\rm E} \equiv Q^*_{\rm E} - q \geqslant 0$ and $\hat{\mu} \equiv \mu/(\tilde{\beta} + \mu)$. For all, $q \geqslant Q^*_{\rm E}$ workers' value functions are invariant equalling $J(Q^*_{\rm E})_{\rm U}$ and $J(Q^*_{\rm E})_{\rm E}$. The interpretation of (6.8) is relatively straightforward. The first term on the RHS represents the expected discounted value to the worker of securing employment before $Q^*_{\rm E}$ is reached (the integral keeps track of the evolution of the worker's type over the relevant interval). The second, term represents the discounted value to the worker of the possibility that he will not secure employment before reaching $Q^*_{\rm E}$ and the value associated with this event. Under Condition R and in the absence of job break-ups ($\delta = 0$), we can fully characterize the re-training problem. Consider,

Proposition 6 (Re-training). *Under Condition R and if $\delta = 0$ there is a re-training equilibrium such that*: (i) *unemployed workers do not re-train themselves and* (ii) *if $q \geqslant Q^*_{\rm E}$ workers are hired and are re-trained on-the-job in the manner described in Lemma* 6 *where $Q^*_{\rm E} = -\ln\{(a_0 - C_0)/a_0\} > 0$.*

With $\delta = 0$ the critical re-training point is defined by: $a_0 \exp[-Q^*_{\rm E}] = a_0 - C_0$, wherein the value of revenues generated by such a worker are exactly equal to the value generated by a new blood worker net of the costs necessary to retrain him.

If an unemployed worker were to re-train himself he would increase his employment valuation, by increasing the size of the 'pie' to be divided between himself and his new employer. However, there are two drawbacks to this strategy. First, the worker incurs all of the re-training costs (and the benefits are shared between the worker and the firm). Second, re-training without a job in hand leads, almost surely, to the relative loss of skill during the time the worker searches for employment. By waiting to re-train once employed, the worker avoids this latter problem completely, and shares the re-training costs with his employer. Proposition 6 says that if there are no job break-ups, these effects outweigh the net benefits to the worker of increasing his own ex ante productivity.

In the presence of re-training, the wage agreement is given by

$$w(q) = \frac{\{a_0 \exp[-q] - v_0\}}{2} + \frac{1}{2} J(q)_{\mathrm{U}}, \tag{6.11}$$

where

$$J(q)_{\mathrm{U}} = b_0 \left\{ a_0(\tilde{\beta} + \frac{1}{2}\mu) + \gamma(a_0 - c_0) \exp\left[-\frac{\tilde{\beta} + \frac{1}{2}\mu}{\gamma} \Delta Q_{\mathrm{E}}^* + q \right] - (\tilde{\beta} + \frac{1}{2}\mu + \gamma) v_0 \exp[-q] \right\} \tag{6.12}$$

with $b_0 \equiv \frac{1}{2}\mu \exp[-q]/[(\tilde{\beta} + \frac{1}{2}\mu)(\tilde{\beta} + \frac{1}{2}\mu + \gamma)]$. The form of the wage agreement (6.11) parallels the wage offer function described in Proposition 2, so that all of our results concerning the new blood effects continue to hold in this environment. Indeed, the only differences are the inclusion of the cost term C_0 and the replacement of the terminal vintage Q by the re-training point Q_{E}^*.

For sufficiently large values of C_0, (in particular if, $a_0 - C_0 - v_0 < 0$), workers for whom $q \geqslant Q_{\mathrm{E}}^*$ are simply not hired by firms and hence not re-trained. In this case, the results are identical to those presented earlier. Finally, we note that although, in the current setup, workers do not re-train themselves, this may not be the case if the re-training cost, takes the more general form $C(q)$, with $C'(q) \geqslant 0$. Here, the re-training cost depends upon the extent to which workers have fallen behind the knowledge frontier. If $C(q)$ rises sufficiently rapidly with q, it is possible that there are circumstances under which unemployed workers, recognizing their imminent demise, may re-train themselves even though they have no job prospect currently in hand.

7. Extensions

In this section, we consider several extensions to the basic model.

7.1. Loss of skills during unemployment and on-the-job learning

In general, the wage agreement is a rather complex function of the parameters.[22] However, if, once again,we ignore job breakups, $\delta = 0$, the wage agreement takes a

[22] More subtly, the wage agreement is in fact a function: $w(\tau_{\mathrm{E}}, \tau_{\mathrm{U}})$ and the value functions in Proposition 1 are *partial* integral equations. The transform to the variable $q(\tau_{\mathrm{E}}, \tau_{\mathrm{U}})$ allows us to write these equations in the form of *ordinary* differential equations in q and to solve the resultant system. Given the implicit presence of partial integral equations, it is perhaps not surprising that the general solution takes on a rather complex form. A complete analysis of this general wage function is available upon request.

rather simple form,

$$w(q) = \left(\frac{\beta + r + \mu + \gamma_{\mathrm{U}}}{\beta + r + \frac{1}{2}\mu + \gamma_{\mathrm{U}}}\right)\frac{a_0}{2}\exp[-q] - \left(\frac{\beta + r + \mu - \gamma_{\mathrm{G}}}{\beta + r + \frac{1}{2}\mu - \gamma_{\mathrm{G}}}\right)\frac{v_0}{2}$$
$$+ \left(\frac{\frac{1}{2}\mu(\gamma_{\mathrm{G}} + \gamma_{\mathrm{U}})}{(\beta + r + \frac{1}{2}\mu + \gamma_{\mathrm{U}})(\beta + r + \frac{1}{2}\mu - \gamma_{\mathrm{G}})}\right)\frac{v_0}{2}$$
$$\times \exp\left[-\frac{\beta + r + \frac{1}{2}\mu - \gamma_{\mathrm{G}}}{\gamma_{\mathrm{G}} + \gamma_{\mathrm{U}}}\Delta Q\right], \tag{7.1}$$

where $Q = \ln[a_0/v_0]$ and $a_0 = \phi(s)K_0/\{r + \beta - \gamma_{\mathrm{E}}\})$. The wage agreement satisfies all of the properties set out in Proposition 2. Furthermore, $w(q)$ increases with γ_{E} and decreases with the skill loss rate γ_{U}, as might be expected. Notice further, that the new blood effect through γ_{G} is quite distinct from the skill loss rate γ_{U}. The reason is that productivity growth, γ_{G}, simultaneously affects both unemployed and employed workers.

7.2. The wage-tenure profile

In this subsection, we are concerned with the wage-tenure profile. Specifically, we intend to compute the flow wage agreement according to employment tenure τ_{E}. The reason that this is of considerable interest is that (i) wages rarely fall (or fall only a little) over the course of a given career (within a particular establishment) and (ii) empirically, the distribution of relative wages according to tenure is ‘hump-shaped’, with workers of moderate tenure earning relatively more than *both* newer entrants *and* older incumbent workers (on this see, for example, Murphy and Welch [27] and Newman and Weiss [28]). Reconciling these two features is an interesting puzzle.

We show that such a pattern arises naturally in an environment in which intergenerational rivalry interacts with on-the-job learning. Intuitively, intergenerational rivalry results, through the new-blood effect, in a negative relationship between wages and employment tenure, while on-the-job learning generates a positive relationship. Below, we demonstrate that a suitable marriage of these two effects can explain the hump-shaped relative wage-tenure patterns. For simplicity, assume no job breakups and no skill losses during unemployment (i.e., $\delta = \gamma_{\mathrm{U}} = 0$). Additionally, we compare wages at a given point in time, conditioning upon workers with the same history and hence type: $q \equiv \gamma_{\mathrm{G}}\tau_{\mathrm{U}} \in [0, Q]$. Consider.

Assumption 4. *The rate of on-the-job learning function* $\Gamma_{\mathrm{E}} : \mathbb{R}_+ \to \mathbb{R}_+$, *is strictly decreasing and continuously differentiable, satisfying*: $\Gamma_{\mathrm{E}}(0) \in (\gamma_{\mathrm{G}}, \beta + r)$ *and* $\lim_{\tau_{\mathrm{E}} \to \infty} \Gamma(\tau_{\mathrm{E}}) \in (0, \gamma_{\mathrm{G}})$.

Assumption 4 implies that on-the-job learning raises human capital levels, but does so at a diminishing rate. Furthermore, the initial rate of on-the-job-learning is relatively rapid, exceeding the (exogenous) rate of public knowledge growth, γ_{G}, and then falls below γ_{G} at a later stage.

Recall that the flow wage, W, received by workers is proportional to the worker's stock of human capital $H(t)$. Since on-the-job-learning now varies with employment tenure, we must modify the human capital equation (2.4) accordingly. The result is,

$$H(t, s, \tau_E, \tau_U) = H_0 \exp[-\gamma_G \tau_E] \exp\left[\int_0^{\tau_E} \Gamma_E(\tau_E{}') \, d\tau_E{}'\right], \tag{7.2}$$

where $H_0 = k(s, t) \exp[-q]$, which is independent of τ_E. By Leibnitz's rule, we have: $dH/d\tau_E = [\Gamma_E(\tau_E) - \gamma_G]H(t)$. Under Assumption 4, it is easily verified that the stock of human capital and hence flow wage possesses a hump-shape over job tenure. The result is summarized in the following proposition.

Proposition 7 (The relative wage-tenure profile). *The equilibrium wage-tenure profile is hump-shaped in the sense that at a given point in time the flow wage increases in his job tenure up to a point and decreases thereafter.*

Intuitively, the presence of this hump-shaped relative wage-tenure profile, described in Proposition 7, is due in essence to the conflicting effects of on-the-job learning and intergenerational rivalry. On the one hand, additional work experience leads to higher productivity and hence higher wages; on the other, the threat of human capital obsolescence from intergenerational competition lowers the wage. With bounded on-the-job learning, the latter effect eventually dominates the former, giving the result reported in the proposition.

With a simple emendation, our model also offers a novel theory of mandatory retirement even with on-going on-the-job training. To see this, assume that μ and s are given and consider,

Assumption 5. *Firms incur a once-off (constant in effective-labor-units) irreversible cost v_0 upon first entering the labor market.*

We can then prove,

Proposition 8 (Mandatory retirement). *Under Assumptions 4 and 5, there exists a finite $\tau_E(q)_M$, corresponding to each worker history q, at which point the worker-firm match endogenously dissolves. At this juncture, firms re-enter the labor market in search of new workers and workers retire to the secondary sector.*

The explanation of this result is simple. Ex ante, at the point of matching, workers receive the greatest expected discounted value of utility by agreeing to separate from the firm when it is efficient for them to do so. In steady-state equilibrium, the free-entry condition (5.1) and the entry-cost structure described in Assumption 5 imply that firms expect to accrue v_0 each time they enter the labor market. Now, under Assumption 4 there is a tenure length, $\tau_E(q)_M$, where separation is optimal. This is because that at fthis point, the expected-discounted continuation value to the firm of retaining the worker is exactly equal to the constant value of terminating the match and searching for workers from younger generations (v_0) (and that the former value declines monotonically with $\tau_E > \tau_E(q)_M$). There are two novel components to this explanation. First, we explain mandatory retirement *not* in terms of a decline in the productivity of the individual, but rather as a consequence of the increase in the human capital possessed by his new-blood competitors. Second, mandatory retirement takes place in a 'memoryless' environment wherein workers do not age (i.e., the flow probability of death is always equal to $\beta > 0$), although emending this latter feature would obviously only strengthen our results.

7.3. Directed search

Up to this point, we have considered random matching within an integrated labor market. It is of interest to check the robustness of our findings in a *directed search* environment, wherein the wage fulfills a familiar allocative role (rather than being determined ex post after matching takes place). The directed-search is developed by Peters [29], Moen [24], Acemoglu and Shimer [2] and Burdett et al. [7] in a variety of goods and labor-market matching games. To best suit our structure, we conceive of the labor market as completely segregated into a series of sub-markets defined by each worker's type, $q \in [0, Q]$. In this context unsuccessful searchers pass, in sequence, through each sub-market $q \in [0, Q]$ until they reach the terminal vintage Q.[23] The motivation for these sub-markets is perhaps best justified on the grounds that employers often advertise for specific skill requirements, such as computer training. The notion that markets are indexed $q \in [0, Q]$, then represents an idealized limit of this process.

We exploit the constant-returns property of the matching technology, assuming that each submarket features the same Beveridge curve relationship: $\eta(\tau) = \eta(\mu(\tau)) \equiv \eta^{SS}(\mu(\tau); m_0)$ described by Lemma 3. Each market is characterized by the pair: $(w(q), \mu(q))$, where $w(q)$ is the wage and $\mu(q)$ is the flow probability that a worker locates a job. A *competitive search equilibrium* is one in which: (i) the pair $(w(q), \mu(q))$ maximizes each worker's valuation, $J(q)_U$ subject to the constraint that firms earn non-negative expected profits: $\Pi_F \geqslant v_0$, (ii) neither firms nor workers have an incentive to switch sub-markets and (iii) there are no profitable opportunities for entry by firms into new sub-markets.[24]

Following similar arguments used to derive the asset value equations in Proposition 1, we show that,

$$J(q)_U = \frac{1}{\gamma_G} \int_q^Q \Bigg\{ \mu(u) w(u) \times \exp\left[-\frac{1}{\gamma_G}(r+\beta)(u-q) + \frac{1}{\gamma_G} \int_q^u (\mu(u'))du' \right] \Bigg\} du, \tag{7.3}$$

$$\Pi(q)_V \equiv \frac{\eta(q)\{a_0 \exp[-q] - w(q)\}}{r + \beta - \gamma_G + \eta(q)}, \tag{7.4}$$

[23] To focus on the essentials, we further assume $\delta = \gamma_E = \gamma_U = 0$ and that $s \geqslant 0$ is exogenous. We again thank an anonymous referee for suggesting this alternative approach to modeling the search environment.

[24] This may be regarded as a generalized version of Moen [24] to a dynamic environment with on-going economic growth (and a continuum of sub-markets).

where $q \in [0, Q]$ and $Q = \ln[a_0/v_0]$. Given that firms are free to enter *any* sub-market we require that: $\Pi(q)_V \geqslant v_0 \geqslant 0$ for each $q \in [0, Q]$.

The optimization problem is one in which the firms in each sub-market q post a wage $w(q)$ and (in effect) choose a service probability, $\mu(q)$, so as to maximize workers' ex ante valuations (7.3), subject to the ex ante non-negative-profit constraint ($\Pi(q)_V \geqslant v_0$). Using (7.4) and the Beveridge curve relationship, the constraint can be used to solve for the wage:

$$w(q) = \{a_0 \exp[-q] - v_0 - v_0(r + \beta - \gamma_G)/\eta(\mu(q))\}. \tag{7.5}$$

Since each firm is negligible in the continuum, the firms' in each sub-market q have no influence upon each worker's *continuation* discount factor: $\Delta(u) \equiv [r + \beta - \int_q^u (\mu(u'))\, du']/\gamma_G$, where $u \in [q, Q]$. It follows that the optimization problem simply entails the point-wise maximization of $\mu(q)w(q)$, with $w(q)$ given by (7.5). The first-order condition is simply $d[\mu w]/d\mu = 0$, or,

$$\eta(\mu^*(q)) = \frac{v_0(r + \beta - \gamma_G)}{a_0 \exp[-q] - v_0}\{1 + \xi(\mu^*(q))\}, \tag{7.6}$$

where $\xi \equiv -(\mu/\eta)(d\eta^{SS}/d\mu) > 0$. Thus, combining (7.5) and (7.6) gives,

$$w^*(q) = \{a_0 \exp[-q] - v_0\}\left\{1 - \frac{1}{1 + \xi(\mu^*(q))}\right\}. \tag{7.7}$$

It is easily checked that the pair: $(w^*(q), \mu^*(q))$, solving the recursive system (7.6) and (7.7), form a competitive-search equilibrium. Since, firms earn precisely v_0 in each sub-market no firm has an incentive to switch markets. Furthermore, since markets are complete on $[0, Q]$, firms have no incentive to form additional sub-markets $q > Q$, since to do so entails a loss. For their part, type $q' \in [0, Q]$ workers have no incentive to search in markets indexed $q < q'$ (since firms are better off holding their vacancies open rather than paying $w(q)$) or in ones indexed $q > q'$, since $J(q)_U$ is *decreasing* in q.

A comparison of (7.7) with the wage agreement (4.1) indicates a remarkable similarity. The key differences are that in (7.7) both $\alpha_0 \exp[-q]$ and v_0 possess identical coefficients and that the capital gain term in (4.1) is no longer present (see Proposition 2). Each of these differences stems from the property of competitive markets to appropriately 'price' workers at the margin (which is not the case under ex post symmetric Nash bargaining). Thus, in (7.7) workers receive a wage proportional to their net (expected discounted) marginal product: $a_0 \exp[-q] - v_0$, while the latter lack-of-capital-gain effect (cf., Proposition 2) reflects the *inability* of type q' workers, in a competitive environment, to extract part of the surplus accruing to other worker types: $q \neq q'$. Even with directed search, the new blood effect appears through two separate channels. First, $\gamma_G > 0$ leads to the terminal search horizon: $T \equiv Q/\gamma_G$, where a higher γ_G shortens the terminal search horizon. Second, in (7.7) notice the term $\mu^*(q)$ which is the *equilibrium* flow contact rate for workers. Crucially, this depends upon the growth rate γ_G. From (7.6), an increase in public knowledge growth (γ_G) leads to a higher worker-contact rate μ^* (provided ξ is a non-decreasing in μ). This finding is similar to the general-equilibrium random-matching result reported in Lemma 5.

The perfectly segregated competitive search-equilibrium is not socially efficient. In particular, a social planner would recognize the transition of unmatched workers as they pass through the sequence of markets $q \in [0, Q]$ and account for the effect of future $\mu(q)$ on the effective discount rate. In this case, the entire sequence of $\{\mu(q)\}$ is chosen to maximize $J(q)_U$ which involves time-varying discounting. Utilizing the fact that $d\Delta \equiv \{(r + \beta - \mu(q))/\gamma_G\}\, du$, we can perform a Uzawa transformation of workers' ex ante value into: $\frac{1}{\gamma_G}\int_{r+\beta}^{\Delta(Q)} \{\mu w \exp[-\Delta]/(\frac{r+\beta-\mu(q)}{\gamma_G})\} d\Delta$, where $w(q)$ is specified as in (7.5). Thus, the optimization becomes a standard constant-discounting optimal control problem, with the first-order condition given by $(r + \beta - \gamma_G)\, \mathrm{d}[\mu w]/d\mu + \mu w = 0$, or,

$$\eta(\mu^{SO}(q)) = \frac{v_0(r + \beta - \gamma_G)}{a_0 \exp[-q] - v_0}\left\{1 + \xi(\mu^{SO}(q))\,\frac{r + \beta - \mu^{SO}(q)}{r + \beta}\right\}, \tag{7.8}$$

where "SO" refers to the social optimum. We can now combine (7.5) and (7.8) to obtain:

$$w^{SO}(q) = \{a_0 \exp[-q] - v_0\}\left\{1 - \frac{1}{1 + \xi(\mu^{SO}(q))\,\frac{r+\beta-\mu^{SO}(q)}{r+\beta}}\right\}. \tag{7.9}$$

By comparing (7.6) and (7.7) with (7.8) and (7.9), we see that the competitive-search equilibrium differs from the social optimum. Given that, $\frac{r+\beta-\mu}{r+\beta} < 1$ and that $d\eta^{SS}/d\mu < 0$, it is straightforward to use (7.6) and (7.8) to show that, provided ξ is a non-decreasing function of μ, $\mu^{SO}(q) > \mu^*(q)$. In this case, it is then immediate from (7.8) that $\xi(\mu^{so}(q))\frac{r+\beta-\mu^{so}(q)}{r+\beta} < \xi(\mu^*(q))$ and from (7.9) that $w^{SO}(q) < w^*(q)$. That is, under directed search the decentralized equilibrium features insufficient firm entry and an excessively *thin* labor market (from the perspective of workers), but is associated with an excessively *high* wage once a match is consummated.

8. Concluding remarks

We believe that the model might admit a number of different extensions and, for brevity, we outline just three of them. First, the assumption that vacancies are identical could be relaxed. Instead, we could also allow for continued improvements in firms' products through an endogenous R&D process (under a monopolistic competition/monopoly setup similar to that in Grossman and Helpman [16] and Aghion and Howitt [3] or a matching framework similar to Acemoglu [1] and Laing et al. [20]). This would lead to a two-side heterogeneity in the search market, with both workers and vacancies of different vintages searching to locate trading partners. One might expect that under these circumstances both types of investment (by workers in schooling and by firms in R&D) would be strategic complements. This would raise a host of issues concerning the appropriate micro-economic policies that might be used to promote growth. Recently, De Long and Summers [12] document interesting evidence supporting the view that investment in new capital equipment is an important determinant of the rate of economic growth.

Finally, we would like to point our that out framework may also be useful to analyze vintage physical capital, along the lines studied by Cooley et al. [11]. In particular, technological advances would render existing tools, machines, and equipment obsolete and thus their rental as well as resale values would be hump-shaped with respect to the corresponding periods of installment. Moreover, the dispersion of capital rental would depend crucially on the degree of severity of technological obsolescence.

Appendix

Proof of Proposition 1 (Asset values). Consider a worker hired by firm at date t' with history (τ_E, τ_U). Let v represent the duration of the current job and $\exp[\gamma_E v]\tilde{W}(t', q)$ represent the flow wage offered to the worker. Then, the flow wage grows at the rate γ_E and at the point of hiring equals $\tilde{W}(t', q)$. Define, $W(q) \equiv \tilde{W}(t', q)/\exp[\gamma_G t']$ as the initial flow wage in effective labor units and let $\tilde{V}(v; t', q) \equiv \exp[\gamma_G t']V(v, q)$ represent the value to the worker of continuing the match, given his experience at the firm v. Finally, let, $\tilde{J}(t, q)_E$ and $\tilde{J}(t, q)_U$ represent workers' valuations. Over small time $h > 0$, the Bellman equation is,

$$\begin{aligned}\tilde{V}(v; t', q) = &\exp[\gamma_E v]\tilde{W}(t', q)h + (1 - rh)(1 - \beta h)\{(1 - \delta h)\tilde{V}(v + h; t', q) \\ &+ \delta h \tilde{J}(t' + v + h, q + \Delta q + dq)_U\},\end{aligned}$$

where $\Delta q = (\gamma_G - \gamma_E)v$ and $dq = (\gamma_G - \gamma_E)h$ describe the evolution of the worker's q at the firm, which is relevant if the current match dissolves, and $\tilde{J}(t + h, q + \Delta q + dq)_U$ is the continuation value in this event. In effective units,

$$\begin{aligned}V(v, q) = &\exp[\gamma_E v]W(q) + (1 - rh)(1 - \beta h)\{(1 - \delta h)V(s + h, q) \\ &+ \delta h \exp[\gamma_G(v + h)]J(q + \Delta q + dq)_U\}.\end{aligned}$$

This relationship holds for all $v \geqslant 0$. Rearranging yields:

$$\begin{aligned}&\{V(v + h, q) - V(v, q)\}/h - \beta' V(v + h, q) \\ &\quad = -\exp[\gamma_E v]W(q) - \delta \exp[\gamma_G v]J(q + \Delta q + dq(h))_U.\end{aligned}$$

where $\beta' \equiv \beta + r + \delta$. Taking the limit as $h \to 0$ gives

$$dV(v, q)/dv - \beta' V(v, q) = -\exp[\gamma_E v]W(q) - \delta \exp[\gamma_G v]J(q + (\gamma_G - \gamma_E)v)_U.$$

For $\gamma_G \neq \gamma_E$, integration then gives,

$$V(v,q) = \lim_{\bar{v}\to\infty} \exp\left[-\int_v^{\bar{v}} \beta' dv\right]\left\{A + \int_v^{\infty} \exp\left[\int_z^{\bar{v}} \beta' dv\right]\{\exp[\gamma_E z]W(q) + \delta \exp[\gamma_G(v+z)]J(q+(\gamma_G-\gamma_E)(v+z))_U\}\right\} dz$$

$$= \frac{\exp[\gamma_E v]W(q)}{\beta'-\gamma_E} + \frac{\delta}{\gamma_G-\gamma_E}\int_q^Q \exp\left[-\frac{\beta'-\gamma_G}{\gamma_G-\gamma_E}(q'-q)\right]J(q')_U\, dq'.$$

The asset value $J(q)_E$ reported in (2.9) is obtained by noting $J(q)_E = V(0,q)$, since $v=0$ corresponds to the point at which the worker is just hired. The other asset values are derived through similar methods. □

Proof of Lemma 1 (The terminal vintage). (i) Let $w(q)$ denote the wage offer function. First, recall that $J(q)_E - J(q)_U = \Pi(q)_F - \Pi_V \geqslant 0$ [see Eq. (2.14)]. Next, define Q such that $a(s,K_0), \exp[-Q] \equiv \Pi_V > 0$. The definition of $\Pi(q)_F$ and the fact that $w(q) \geqslant 0$ imply $a(s,K_0)\exp[-q] - \Pi_V \geqslant \Pi(Q)_F - \Pi_V$. It follows then that $0 > a(s,K_0)\exp[-q] - \Pi_V \geqslant \Pi(q)_F - \Pi_V$ for all $q > Q$, implying that after Q the wage is negative. Note that $w(Q) = 0$, since $0 \equiv a(s,K_0)\exp[-Q] - \Pi_V = \Pi(Q)_F - \Pi_V \geqslant 0$. Uniqueness of Q is proved below.

(ii)–(iii) The reported derivatives are obtained easily by totally differentiating $a(s,K_0)\exp[-Q] \equiv \Pi_V$. □

Proof of Lemma 2 (The distribution function $F(q)_U$). (i) Supports: If $\gamma_E \leqslant \gamma_G$, then (2.5) indicates that $q \geqslant 0$. Workers search for employment only if, $q \leqslant Q$ (Lemma 1), so that: $q \in [0,Q]$. Alternatively, if $\gamma_E > \gamma_G$, then: $\lim_{\tau_E\to\infty} q(\tau_E,\tau_U) = -\infty$. Then from Lemma 1, $q \in (-\infty, Q]$.

(ii) The definition of the cumulative probability distribution function, is $F(q)_U \equiv \int_0^q g(q')_U dq' / \int_0^Q g(q')_U dq'$. Similar remarks apply to $F(q)_E$.

(iii) The flow birth rate of workers is β. Let $g(q)_U$ and $g(q)_E$ represent, respectively, the *population* densities of unemployed and employed workers with type q. We have

$$g(0)_U = 1 - g(0)_E = \beta, \tag{A.1}$$

since all new entrant workers are initially unemployed. For small time period $h > 0$, we have

$$g(q+\delta q)_E = (1-\beta h)(1-\delta h)g(q)_E + \mu h(1-\beta h)g(q)_U,$$

indicating that the population density of employed workers $q+\delta q$ equals the population of q workers who do not die $(1-\beta h)$ and do not lose their job $(1-\delta h)$ plus unemployed workers who survive $(1-\beta h)$ and find a job (μh). Re-arranging this expression yields:

$$\frac{\{g(q+\delta q)_E - dg(q)_E\}}{\delta q}\frac{\delta q}{h} = -(\beta+\delta)g(q)_E + \mu g(q)_U.$$

The limit as $h \to 0$ then yields the following differential equation, (provided $(\gamma_G - \gamma_E) \neq 0$),

$$dg(q)_E/dq = -\frac{(\beta+\delta)}{(\gamma_G - \gamma_E)} g(q)_E + \frac{\mu}{(\gamma_G - \gamma_E)} g(q)_U, \tag{A.2}$$

where the following result is used $\lim_{h\to 0} \delta q/h = (\gamma_G - \gamma_E)$. Similarly,

$$dg(q)_U/dq = \frac{\delta}{(\gamma_G + \gamma_U)} g(q)_E - \frac{(\beta+\mu)}{(\gamma_G + \gamma_U)} g(q)_U. \tag{A.3}$$

Together, (A.2) and (A.3) represent a simple pair of first-order linear homogeneous differential equations. The solution form is

$$g(\tau)_E = A_1 \exp[\lambda_1 q] + A_2 \exp[\lambda_2 q], \tag{A.4}$$

$$g(\tau)_U = A_3 \exp[\lambda_1 q] + A_4 \exp[\lambda_2 q], \tag{A.5}$$

where $A_1, \ldots, A_4$ are integration constants and λ_1 and λ_2 are roots of the characteristic equation,

$$\lambda^2 + \lambda\left(\frac{\beta+\delta}{\gamma_G - \gamma_E} + \frac{\beta+\mu}{\gamma_G + \gamma_U}\right) + \frac{\beta(\beta+\delta+\mu)}{(\gamma_G - \gamma_E)(\gamma_G + \gamma_U)} = 0. \tag{A.6}$$

Using (A.4) and (A.5), in conjunction with the boundary conditions (A.1), yield: $0 = A_1 + A_2$ and $\beta = A_3 + A_4$. Differentiating (A.4) with respect to q, in conjunction with λ_1 and λ_2 and $A_1 = -A_2$, yields,

$$dg(q)_E/dq = \lambda_1 A_1 \exp[\lambda_1 q] - A_1 \lambda_2 \exp[\lambda_2 q].$$

Furthermore, using solution (A.4) and (A.5) and $A_4 = \beta - A_3$ in (A.2) gives,

$$\begin{aligned} dg(q)_E/dq = &\left(A_3 \frac{\mu}{\gamma_G - \gamma_E} - \frac{\beta+\delta}{\gamma_G - \gamma_E} A_1\right)\exp[\lambda_1 q] \\ &+ \left(\frac{\mu}{\gamma_G - \gamma_E}(\beta - A_3) + \frac{\beta+\delta}{\gamma_G - \gamma_E} A_1\right)\exp[\lambda_2 q]. \end{aligned}$$

A comparison of coefficients between these two equations gives

$$A_1 = \frac{\mu\beta}{(\gamma_G - \gamma_E)(\lambda_1 - \lambda_2)} \quad \text{and} \quad A_3 = \frac{\beta(r_1(\gamma_G - \gamma_E) + \beta + \delta)}{(\gamma_G - \gamma_E)(\lambda_1 - \lambda_2)}. \tag{A.7}$$

Using these findings, $A_1 = -A_2$, and the result that $A_4 = \beta - A_3$ gives the results reported in the text.

Proof of Corollary 1 (Distribution functions under $\gamma_E = \gamma_U = 0$). Since (A.6) factorizes as: $\lambda_1 = -\beta/\gamma_G$ and $\lambda_2 = -(\beta+\delta+\mu)/\gamma_G$, (A.7) gives,

$$g(q)_U = \frac{\beta\delta}{\delta+\mu}\exp[-\beta q/\gamma_G] + \frac{\beta\mu}{\delta+\mu}\exp[-(\beta+\delta+\mu)q/\gamma_G]. \tag{A.8}$$

Integrating (A.8) with respect to q yields the expression in the text. Similar remarks apply to $g(q)_E$. □

Proof of Lemma 3 (The *SS* locus). (i) *Limiting properties*: the limiting results follow directly from the Inada conditions.

(ii)–(iii) *Derivatives with respect to* μ, *and* m_0: Using the constant returns to scale property of $M(\cdot)$, in conjunction with (3.10), write $\eta = m_0 M(\eta/\mu, 1)$. Totally differentiating this expression yields, $(\mu - m_0 M_U)\, d\eta + m_0 M_U(\eta/\mu)\, d\mu - \mu M(\eta/\mu, 1)\, dm_0 = 0$, implying that the signs of the derivatives hinge upon: $(\mu - m_0 M_U)$. From the Euler theorem, using (17), we have

$$\mu U = m_0 M(U, V) = m_0[M_U(\eta/\mu, 1)U + M_V(\eta/\mu, 1)V]$$

which implies $\mu - m_0 M_U = m_0 M_V(\mu/\eta) > 0$, proving the result. □

Proof of Proposition 2 (The wage agreement, $w(q;.)$). (a) *Determination of* $w(q;.)$. Differentiating asset values (2.9) and (2.11) with respect to q gives

$$\dot{J}_E = \dot{w} + (\tilde{\beta} + \delta)\gamma_G^{-1}(J_E - w) - (\delta/\gamma_G)J_U, \tag{A.9}$$

$$\dot{J}_U = -\frac{\mu}{\gamma_G} J_E + (\beta')\gamma_G^{-1} J_U. \tag{A.10}$$

where $\dot{x} \equiv \partial x/\partial q$, $\tilde{\beta} \equiv r + \beta - \gamma_G$, and the argument q is suppressed from w and the asset value equations. The Nash bargaining (2.14) condition implies that,

$$J_E \equiv a_0 \exp[-q] - w + J_U - \Pi_V, \tag{A.11}$$

$$\dot{J}_E \equiv -a_0 \exp[-q] - \dot{w} + \dot{J}_U. \tag{A.12}$$

Substituting (A.11) and (A.12) into (A.9) gives,

$$-2\dot{w} + \dot{J}_U = -2\left(\frac{\tilde{\beta} + \delta}{\gamma_G}\right) w + \left(\frac{\tilde{\beta}}{\gamma_G}\right) J_U + \left(\frac{\tilde{\beta} + \delta + \gamma_G}{\gamma_G}\right) z - \left(\frac{\tilde{\beta} + \delta}{\gamma_G}\right) \Pi_V, \tag{A.13}$$

where $z(q) \equiv a_0 \exp[-q]$ and

$$\dot{z} = -z. \tag{A.14}$$

Likewise, using (A.11) in (A.10) gives,

$$\dot{J}_U = (\mu/\gamma_G)w + (\tilde{\beta}/\gamma_G)J_U - (\mu/\gamma_G)z + (\mu/\gamma_G)\Pi_V. \tag{A.15}$$

Together, (A.13), (A.14), and (A.10) represent a system of linear first-order differential equations with constant coefficients in the variables: w, J_U, and z. The system may be written,

$$\begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \dot{w} \\ \dot{J}_U \\ \dot{z} \end{pmatrix} = \begin{bmatrix} -2(\frac{\tilde{\beta}+\delta}{\gamma_G}) & (\frac{\tilde{\beta}}{\gamma_G}) & (\frac{\tilde{\beta}+\delta+\gamma_G}{\gamma_G}) \\ (\mu/\gamma_G) & (\tilde{\beta}/\gamma_G) & -(\mu/\gamma_G) \\ 0 & 0 & -1 \end{bmatrix} \begin{pmatrix} w \\ J_U \\ z \end{pmatrix} + \begin{pmatrix} -(\frac{\tilde{\beta}+\delta}{\gamma_G})\Pi_V \\ (\mu/\gamma_G)\Pi_V \\ 0 \end{pmatrix}. \tag{A.16}$$

The solution to (A.16) is of the form:

$$w = B_1 \exp[\lambda_1 q] + B_2 \exp[\lambda_2 q] + B_3 \exp[\lambda_3 q] + w_p, \tag{A.17}$$

$$J_U = B_4 \exp[\lambda_1 q] + B_5 \exp[\lambda_2 q] + B_6 \exp[\lambda_3 q] + J_{Up}, \tag{A.18}$$

$$z = B_7 \exp[\lambda_1 q] + B_8 \exp[\lambda_2 q] + B_9 \exp[\lambda_3 q] + z_p, \tag{A.19}$$

where $B_1, \dots, B_9$ are as-yet unknown coefficients, w_p, J_{Up}, and z_p are particular solutions and λ_1, λ_2 and λ_3 are roots of the characteristic equation,

$$(\lambda + 1)\left(\lambda^2 - \lambda\left(\frac{\tilde{\beta} + \delta}{\gamma_G} + \frac{\tilde{\beta} + \frac{1}{2}\mu}{\gamma_G}\right) + \frac{\tilde{\beta}(\tilde{\beta} + \delta + \frac{1}{2}\mu)}{\gamma_G^2}\right) = 0,$$

which possesses the distinct (real) roots: $\lambda_1 = -1$, $\lambda_2 = -\tilde{\beta}/\gamma_G$, and $\lambda_3 = (\tilde{\beta} + \delta + \mu/2)/\gamma_G$. The coefficients, $B_1, \dots, B_9$ are determined, after lengthy calculations, by first differentiating (A.17)–(A.19) with respect to q, using the values of the roots, the boundary conditions $w(Q) = J(Q)_U$. Once this is done, the solutions (A.17)–(A.19) are then substituted in (A.13) and (A.15). The coefficients A_i are then solved by comparing their values in each of the above equations. Finally, the particular solutions are determined by setting $\dot{w} = \dot{J}_U = \dot{z} = 0$ in (A.16) and solving the resulting system of equations. The wage equation reported in the text is then determined from (A.17).

(b) *Properties of the wage agreement*. Denoting $\beta' = (r + \delta + \beta)$, the wage agreement may be written as

$$\begin{aligned} 2w(q)\exp[q]/a(s, K_0) = & \frac{\beta' + \mu}{\beta' + \frac{1}{2}\mu} + \frac{\gamma_G(\frac{1}{2}\mu)}{(\beta' + \frac{1}{2}\mu)(\beta' + \frac{1}{2}\mu - \gamma_G)} \exp\left[-\frac{(\beta' + \frac{1}{2}\mu)}{\gamma_G}\Delta Q\right] \\ & - \frac{\beta' + \mu - \gamma_G}{\beta' + \frac{1}{2}\mu - \gamma_G} \exp[-\Delta Q]. \end{aligned} \tag{A.20}$$

(i) Differentiating this expression with respect to q gives,

$$\begin{aligned} 2(\dot{w} + w)\exp[q]/a(s, K_0) &= (\beta' + \mu/2 - \gamma_G)^{-1} \exp[-\Delta Q] \\ &\quad \times \{(\mu/2)\exp[-(\beta' + \mu/2 - \gamma_G)/\gamma_G] - (\beta' + \mu - \gamma_G)\} \\ &< (\beta' + \mu/2 - \gamma_G)^{-1} \exp[-\Delta Q]\{\mu/2 - (\beta' + \mu - \gamma_G)\} \\ &= (\beta' + \mu/2 - \gamma_G)^{-1} \exp[-\Delta Q]\{\gamma_G - \beta' - \mu/2\} < 0, \end{aligned}$$

where the inequalities follow from the condition $\beta + r + \delta - \gamma_g > 0$. Thus, $\dot{w}/w < -1$ as claimed. By definition, $w(Q) = 0$. Furthermore, $w(q)$ is strictly decreasing in q, which implies that Q is unique (thus completing the proof of Lemma 1).

(ii) Derivatives,

(a) $\partial w/\partial s$: Differentiating (A.20) with respect to s, gives, after rearrangement:

$$\partial w/\partial s = (\partial a(s, K_0)/\partial s) w(q)/a_0 + [\partial w/\partial Q](\partial Q/\partial s) > 0,$$

where the result follows from, $\partial a(s, K_0)/\partial s$, Lemma 1, and $[\partial w/\partial Q](2/a_0) = (\Delta_2)^{-1}\{(\beta' - \gamma_G) + \mu(1 - (1/2)\exp[-(\Delta_2 \Delta Q)/\gamma_G])\}\exp[-\Delta Q] > 0$

(b) $\partial w/\partial \mu$: Differentiating (A.20) with respect to μ gives:

$$
\begin{aligned}
&4\partial(w(q)/\partial\mu)\exp[q]/a(s, K_0)\\
&\quad = \frac{\beta'}{\Delta_1^2} - \frac{\beta' - \gamma_G}{\Delta_2^2}\exp[-\Delta Q] + (\Delta_1\Delta_2)^2(\gamma_G(\Delta_1\Delta_2 - (\mu/2)(\Delta_1 + \Delta_2))\\
&\quad - \Delta_1\Delta_2(\mu/2)\Delta Q)\exp[-\Delta_1(\gamma_G^{-1})\Delta Q], \qquad \text{(A.21)}
\end{aligned}
$$

where $\Delta_1 \equiv \beta' + \mu/2$ and $\Delta_2 \equiv \Delta_1 - \gamma_G$. In order to prove the result, treat $\Delta Q \in [0, \infty)$ as a choice variable and consider the problem of minimizing the right-hand-side of (A.21) with respect to it. The result follows if the minimized expression is positive. The first-order conditions for an interior minimum are,

$$
\begin{aligned}
&(\gamma_G(\Delta_1\Delta_2 - (\mu/2)(\Delta_1 + \Delta_2)) - \Delta_1\Delta_2(\mu/2)\Delta Q)\exp[-\Delta_1(\gamma_G^{-1})\Delta Q]\\
&\quad = (\gamma_G(\Delta_1)(\beta' - \gamma_G)\exp[-\Delta Q] - \gamma_G(\mu\Delta_2/2))\exp[-\Delta_1(\gamma_G^{-1})\Delta Q].
\end{aligned}
$$

If the solution is on the boundary, $\Delta Q = 0$, the result is trivial. Substituting this result in (A.21) gives,

$$
Sgn\{\partial w(q)/\partial\mu)\} = Sgn\{\beta'\Delta_2^2 - (\beta' - \gamma_G)\Delta_1\Delta_2\exp[-\Delta Q] - \gamma_G(\mu/2)\Delta_2\exp[-\Delta_1(\gamma_G^{-1})\Delta Q]\}.
$$

But, since $\Delta Q > 0$ and $\Delta_1 > 0$,

$$
\begin{aligned}
&\beta'\Delta_2^2 - (\beta' - \gamma_G)\Delta_1\Delta_2\exp[-\Delta Q] - \gamma_G(\mu/2)\Delta_2\exp[-\Delta_1(\gamma_G^{-1})\Delta Q]\\
&\quad > Sgn\{\beta'\Delta_2^2 - (\beta' - \gamma_G)\Delta_1\Delta_2 - \gamma_G(\mu/2)\Delta\} = 0
\end{aligned}
$$

establishing the result. The derivative for γ_G and Π_V are proved through similar methods.

(iii) The results for workers' real income are proved through methods similar to those for μ above and are consequently not reported. □

Proof of Corollary 2 (The wage under $\gamma_G \to 0$). Recall that $Q \equiv \ln[a_0/\Pi_V]$. Taking limits in (4.1) yields $w(0) = \frac{(\beta'+\mu)}{(\beta'+\frac{1}{2}\mu)}\frac{a_0 - \Pi_V}{2} > 0$. □

The Variance of the Wage Distribution According to the Date of Employment t': Let $E(w)$ and $E(w^2)$ denote the first and second moment of the wage distribution, respectively. Using (4.1), tedious calculations show:

$$
\begin{aligned}
E(w) = \int_0^T w(\tau)g(\tau)\,d\tau = W_0\bigg\{&\frac{1}{\mu + \beta + \gamma_G}(1 - \exp[-(\mu + \beta + \gamma_G)T])\\
&- \frac{R_1\exp[-\gamma_G T]}{\mu + \beta}(1 - \exp[-(\mu + \beta)T])\\
&+ \frac{R_0}{\mu + \beta - \theta_0}(1 - \exp[-(\mu + \beta - \theta_0)T])\bigg\},
\end{aligned}
$$

$$
\begin{aligned}
E(w^2) &= \int_0^T \{w(\tau)^2\} g(\tau)\,\mathrm{d}\tau \\
&= (W_0)^2 \frac{1-\exp[-(\mu+\beta)T]}{\mu+\beta} \left\{ \frac{1}{\mu+\beta+2\gamma_G}(1-\exp[-(\mu+\beta+2\gamma_G)T]) \right. \\
&\quad + \frac{(R_1)^2}{\mu+\beta}\exp[-2\gamma_G T](1-\exp[-(\mu+\beta)T]) \\
&\quad + \frac{(R_0)^2}{\mu+\beta-2\theta_0}(1-\exp[-(\mu+\beta-2\theta_0)T]) \\
&\quad - \frac{2R_1}{\mu+\beta+\gamma_G}\exp[-\gamma_G T](1-\exp[-(\mu+\beta+\gamma_G)T]) \\
&\quad + \frac{2R_0}{\mu+\beta+\gamma_G-\theta_0}(1-\exp[-(\mu+\beta+\gamma_G-\theta_0)T]) \\
&\quad \left. - \frac{2R_0R_1}{\mu+\beta-\theta_0}\exp[-\gamma_G T][1-\exp(-(\mu+\beta-\theta_0)T)] \right\},
\end{aligned}
$$

where $W_0 = \frac{\hat{w}a(\mu+\beta)}{2(1-\exp[-(\mu+\beta)T])}$. The variance of the wage, $Var[w] = E(w^2) - (E(w))^2$, then follows.

Proof of Lemma 4 (The *EE* locus). Under the free entry Condition, $v_0 = \frac{\eta\hat{\Pi}_F}{\eta+r-\gamma_G}$, where $\hat{\Pi}_F \equiv \int_0^Q \Pi(q)_F f(q)_U \, dq$ and $f(q)_U$ is the density reported in Lemma 2. This equation defines a function $\eta = \eta^{EE}(\mu, s; \gamma_G)$, since the Nash bargaining condition and Condition N implies that $\Pi(q)_F > v_0$ over a set of $q \in [0, Q]$ with strictly positive measure. The proofs of the various assertions made in Lemma 4 requires the evaluation of $\hat{\Pi}_F$. By definition, $\Pi(q)_F = a(s, K_0) - w(q)$. From Proposition 2 we have

$$
\begin{aligned}
\Pi(q)_F =& \frac{a_0}{2}\frac{\beta'}{\beta'+\frac{1}{2}\mu}\exp[-q] + \frac{\beta'+\mu-\gamma_G}{\beta'+\frac{1}{2}\mu-\gamma_G}\frac{v_0}{2} \\
&- \frac{\frac{1}{2}\mu\gamma_G}{(\beta'+\frac{1}{2}\mu)(\beta'+\frac{1}{2}\mu-\gamma_G)}\frac{v_0}{2}\exp\left[-\frac{\beta'+\frac{1}{2}\mu-\gamma_G}{\gamma_G}\Delta Q\right].
\end{aligned}
\tag{A.22}
$$

Integrating (A.22) by parts gives

$$
\hat{\Pi}_F = v_0 - \int_0^Q \dot{\Pi}(q)_F F(q)\, dq, \tag{A.23}
$$

where $\Pi(Q)_F F(Q) = v_0$ and $\Pi(0)_F F(0) = 0$. Differentiating $\Pi(q)_F$ with respect to q yields

$$
\dot{\Pi}(q)_F = -\frac{\beta'}{\beta'+\frac{1}{2}\mu}\frac{a_0}{2}\exp[-q] - \frac{\frac{1}{2}\mu}{\beta'+\frac{1}{2}\mu}\frac{v_0}{2}\exp\left[-\frac{(\beta'+\frac{1}{2}\mu-\gamma_G)}{\gamma_G}\Delta Q\right],
$$

where $\Delta Q \equiv Q - q$ and $\beta' \equiv \beta + \delta + r$.

(i) *Limiting properties*
The application of standard methods reveals

$$\lim_{\mu\to 0}(\hat{\Pi}_{\rm F}-v_0)=\frac{a_0}{2}\left\{(1-\exp[-Q])-\frac{\gamma_{\rm G}}{\gamma_{\rm G}+\beta}\left(1-\exp\left[-\left(\frac{\gamma_{\rm G}+\beta}{\beta}\right)Q\right]\right)\right\}$$
$$>\frac{a_0}{2}(1-\exp[-Q])\left\{-\frac{\beta}{\gamma_{\rm G}+\beta}\right\}>0.$$

But then, $\eta_0\equiv\lim_{\mu\to 0}(\frac{v_0(r-\gamma_{\rm G})}{\hat{\Pi}_{\rm F}-v_0})>0$. As $\mu\to\infty$, the distribution function $F(q)$ converges uniformly to,

$$F^*(q)\equiv\lim_{\mu\to\infty}F(q)=\frac{1-\exp[-(\beta/\gamma_{\rm G})q]}{1-\exp[-(\beta/\gamma_{\rm G})Q]}>0$$

for all $q>0$. However, the expression, $\dot{\Pi}_{\rm F}$ converges uniformly to zero over the interval $[0,Q]$ as, $\lim_{\mu\to\infty}\dot{\Pi}_{\rm F}=0$, which implies that:

$$\lim_{\mu\to\infty}\{\hat{\Pi}_{\rm F}-v_0\}=\lim_{\mu\to\infty}\int_0^Q\dot{\Pi}(q)_{\rm F}F(q)\,{\rm d}q=0;\quad \eta_1\equiv\lim_{\mu\to\infty}\frac{v_0(r-\gamma_{\rm G})}{\hat{\Pi}_{\rm F}-v_0}=\infty.$$

(ii) *Derivative with respect to* μ
We have,

$$\hat{\Pi}^*_{\rm F}-v_0\equiv\lim_{\gamma_{\rm G}\to 0}\{\hat{\Pi}_{\rm F}-v_0\}=\frac{\beta'}{\beta'+\frac{1}{2}\mu}\left(\frac{\alpha_0-v_0}{2}\right),$$

hence, $\partial\{\hat{\Pi}^*_{\rm F}-v_0\}/\partial\mu<0$ and $\partial\eta^{EE}/\partial\mu>0$ as claimed for some half-open interval, $\gamma_{\rm G}\in[0,\gamma_{\rm G})$

(iii) *Derivative with respect to* $a_0\equiv a(s,K_0)$.

Given that, $\partial a_0/\partial s>0$ and $\partial a_0/\partial K_0>0$, we can evaluate the effects of s and K_0 upon η^{EE} through knowledge of the sign of $\partial\eta^{EE}/\partial a_0$ alone. The definition $a_0\exp[-Q]\equiv v_0$, allows us to write

$$\dot{\Pi}(q)_{\rm F}=-\frac{\beta'}{\beta'+\frac{1}{2}\mu}\frac{v_0}{2}\exp[-\Delta Q]-\frac{\frac{1}{2}\mu}{\beta'+\frac{1}{2}\mu}\frac{v_0}{2}\exp\left[-\frac{(\beta'+\frac{1}{2}\mu-\gamma_{\rm G})}{\gamma_{\rm G}}\Delta Q\right].\quad\text{(A.24)}$$

Eq. (A.23) may be rewritten as

$$\hat{\Pi}_{\rm F}-v_0\equiv-\int_0^Q\dot{\Pi}(-Q+q)_{\rm F}F(q;Q)\,dq.$$

Consider the change of variable, $\Delta Q\equiv Q-q\geqslant 0$. Then,

$$\hat{\Pi}_{\rm F}-v_0\equiv\int_0^Q\{-\dot{\Pi}(\Delta Q)_{\rm F}\}F(Q-\Delta Q;Q)\,d(\Delta Q).\quad\text{(A.25)}$$

Notice that, as written, (A.24) does not depend upon a_0 and that in the integral in (A.25), $-\dot{\Pi}_{\rm F}>0$. Consequently, from Leibnitz rule,

$$\partial\{\hat{\Pi}_{\rm F}-v_0\}/\partial a_0=-\dot{\Pi}(0)_{\rm F}F(0;Q)$$
$$+\int_0^Q\{-\dot{\Pi}(\Delta Q)_{\rm F}\}\left(\frac{\partial\{F(Q-\Delta Q;Q)\}}{\partial a_0}\right)d(\Delta Q).$$

Since F is the cumulative distribution function of q, $F(0;Q)=0$, so that the sign depends upon $\frac{\partial\{F(Q-\Delta Q;Q)\}}{\partial a_0}$. Writing the cumulative distribution function F in terms of the underlying population distributions (Lemma 2) yields $F=\frac{G(Q-\Delta Q)}{G(Q)}$. Differentiating this with respect to a_0 (noting that, $Q\equiv \ln[a_0/v_0]>0$) gives,

$$\frac{\partial\{F(Q-\Delta Q;Q)\}}{\partial a_0}=\left(\frac{1}{a_0}\right)\left[\frac{G'[Q-\Delta Q]G(Q)-G'[Q]G[Q-\Delta Q]}{G[Q]^2}\right]. \tag{A.26}$$

Evaluating (A.26) gives,

$$\begin{aligned} sgn&\left(\frac{\partial\{F(Q-\Delta Q;Q)\}}{\partial a_0}\right)\\ &= sgn\,\delta\exp\left[-\frac{\beta}{\gamma_{\mathrm{G}}}Q\right]\left(\exp\left[\frac{\beta}{\gamma_{\mathrm{G}}}\Delta Q\right]\right)-1\\ &\quad+\mu\exp\left[-\frac{(\beta+\delta+\mu)}{\gamma_{\mathrm{G}}}Q\right]\left(\exp\left[\frac{(\beta+\delta+\mu)}{\gamma_{\mathrm{G}}}\Delta Q\right]-1\right)\geqslant 0,\end{aligned}$$

as $\Delta Q\geqslant 0$. It follows that $\partial\{\hat{\Pi}_{\mathrm{F}}-v_0\}/\partial a_0>0$ since, $\frac{\partial\{F(Q-\Delta Q;Q)\}}{\partial a_0}=0$ only on a subset of $[0,Q]$ of zero measure (namely, $\Delta Q=0$) and $\frac{\partial\{F(Q-\Delta Q;Q)\}}{\partial a_0}>0$ otherwise. Consequently,

$$\partial\eta^{EE}/\partial a_0=-\frac{\partial\{\hat{\Pi}_{\mathrm{F}}-v_0\}}{\partial a_0}\left(\frac{\eta}{(\hat{\Pi}_{\mathrm{F}}-v_0)}\right)<0.$$

(iv) *Derivative with respect to* γ_{G}. From the definition of the *EE* locus we have,

$$\lim_{\gamma_{\mathrm{G}}\to 0}\frac{\partial\eta^{EE}}{\partial\gamma_{\mathrm{G}}}=\left(\frac{r(\partial\{\hat{\Pi}^*_{\mathrm{F}}\}/\partial\gamma_{\mathrm{G}})-(\hat{\Pi}^*_{\mathrm{F}}-v_0)}{(\hat{\Pi}^*_{\mathrm{F}}-v_0)^2}\right),$$

where $\lim_{\gamma_{\mathrm{G}}\to 0}\hat{\Pi}_{\mathrm{F}}\equiv\hat{\Pi}^*_{\mathrm{F}}$ and $\partial\hat{\Pi}^*_{\mathrm{F}}/\partial\gamma_{\mathrm{G}}\equiv\lim_{\gamma_{\mathrm{G}}\to 0}(\partial\hat{\Pi}_{\mathrm{F}}/\partial\gamma_{\mathrm{G}})$. Lengthy calculations give,

$$Sgn\left\{\lim_{\gamma_{\mathrm{G}}\to 0}\frac{\partial\eta^{EE}}{\partial\gamma_{\mathrm{G}}}\right\}=sgn\{\chi(\mu)\},$$

where $\chi(\mu)\equiv\left(\frac{1}{2(\beta'+\frac{1}{2}\mu)}\right)\left\{-\left[\frac{\beta' r\{(\beta+\delta)^2+\delta\mu\}}{\beta(\beta+\delta)(\beta+\delta+\mu)}\right]a_0+r(\mu/2)v_0-\beta'(a_0-v_0)\right\}$. We have

$$\lim_{\mu\to 0}\chi(\mu)=-\frac{1}{2}\left(\frac{r}{\beta}\right)a_0-\frac{1}{2}(a_0-v_0)<0;\quad \lim_{\mu\to\infty}\chi(\mu)=rv_0/2>0.$$

The above limits imply that the (continuous) function $\chi(\mu)$ possesses at least one root, $\bar{\mu}:\chi(\bar{\mu})\equiv 0$. That the root $\bar{\mu}$ is unique is seen by differentiating $\chi(\mu)$ around *any* $\chi(\bar{\mu})\equiv 0$, to yield:

$$Sgn\left\{\left.\frac{\partial\chi(\mu)}{\partial\mu}\right|_{\mu=\bar{\mu}}\right\}=sgn\left\{\left(\frac{a_0\beta' r\beta(\beta+\delta)}{(\beta+\delta+\bar{\mu})^2}\right)+(1/2)(rv_0)\right\}>0.$$

The root is unique since $\lim_{\mu\to 0}\chi(\mu)<0$ and $Sgn\{\frac{\partial\chi(\mu)}{\partial\mu}|_{\mu=\bar{\mu}}\}>0$ imply a unique turning point. $\square$

Proof of Proposition 3 (Steady-state equilibrium). Existence is immediate. The *EE* and *SS* loci are continuous and, given their limiting properties described in Lemmas 3 and 4, must cross at least once. The point of intersection determines a pair (μ^*, η^*) consistent with the free-entry and steady-state conditions. Once μ^* and η^* are determined, the other endogenous variables can then be calculated recursively. For small values of γ_G, the locus *EE* is monotonically increasing and can thus intersect the *SS* locus, which is monotonically decreasing, at most once. $\square$

Proof of Lemma 5 (Optimal schooling). From Proposition 1 the asset value of a job seeker is, for each $q\in[0, Q]$,

$$\begin{aligned}J(q)_U \equiv{}& \frac{\beta'\mu}{(\beta'+\frac{1}{2}\mu)(\beta'-\delta)}\frac{a_0}{2}\exp[-q]\\ &+\frac{\gamma_G(\frac{1}{2}\mu^2)}{(\beta'+\frac{1}{2}\mu)(\beta'+\frac{1}{2}\mu-\gamma_G)(\delta+\frac{1}{2}\mu)}\frac{v_0}{2}\exp\left[-\frac{(\beta'+\frac{1}{2}\mu-\gamma_G)}{\gamma_G}\Delta Q\right]\\ &+\frac{\delta\gamma_G\mu}{(\beta'-\delta)(\beta'-\delta-\gamma_G)(\delta+\frac{1}{2}\mu)}\frac{v_0}{2}\exp\left[-\frac{(\beta'-\delta-\gamma_G)}{\gamma_G}\Delta Q\right]\\ &-\frac{(\beta'-\gamma_G)\mu}{(\beta'-\delta-\gamma_G)(\beta'+\frac{1}{2}\mu-\gamma_G)}\frac{v_0}{2},\end{aligned}\tag{A.27}$$

where $\beta'\equiv\beta+r+\delta$, $\Delta Q\equiv Q-q$, $Q\equiv\ln(a_0/v_0)$, and $a_0\equiv a(s,K_0)$. At the point of their entry into the labor market, $q=0$. Solving: $\max_{s\geqslant 0}\{J(0)_U-c(s)\}$ by differentiating (A.27) with respect to s yields:

$$\begin{aligned}&\frac{\beta'\mu}{(\beta'+\frac{1}{2}\mu)(\beta'-\delta)}\frac{a_0}{2}(a_s/a_0)\\ &-\frac{(\frac{1}{2}\mu^2)}{(\beta'+\frac{1}{2}\mu)(\delta+\frac{1}{2}\mu)}\frac{v_0}{2}\exp\left[-\frac{(\beta'+\frac{1}{2}\mu-\gamma_G)}{\gamma_G}Q\right](a_s/a_0)\\ &-\frac{\delta\mu}{(\beta'-\delta)(\delta+\frac{1}{2}\mu)}\frac{v_0}{2}\exp\left[-\frac{(\beta'-\delta-\gamma_G)}{\gamma_G}Q\right](a_s/a_0)-c'(s)=0,\end{aligned}\tag{A.28}$$

where $a_s\equiv\partial a(s,K_0)/\partial s>0$. That s possesses an interior solution, provided $\mu>0$, follows, since $a_0\equiv v_0\exp[Q]>v_0$, $\beta'>\gamma_G$, and $\beta'-\gamma_G-\delta\equiv r+\beta-\gamma_G>0$, implying that the left-hand side of (A.28) strictly exceeds the sum of the coefficients:

$$\frac{\beta'\mu}{(\beta'+\frac{1}{2}\mu)(\beta'-\delta)}-\frac{(\frac{1}{2}\mu^2)}{(\beta'+\frac{1}{2}\mu)(\delta+\frac{1}{2}\mu)}-\frac{\delta\mu}{(\beta'-\delta)(\delta+\frac{1}{2}\mu)}=0,$$

implying $s>0$. Furthermore, since $a(s,K_0)$ and $c(s)$ are, respectively, strictly concave and convex, the maximizing value of s is bounded above. Consequently, there exists (at least one) interior maximum. Differentiating (A.28) with respect to s around *any* given stationary point yields:

$$\frac{a_{ss}}{a_0}\frac{1}{a_s}c'(s) - c''(s) + \left(\frac{a_s}{a_0}\right)^2\left[\frac{\mu}{\delta+\frac{1}{2}\mu}\right]$$
$$\times\frac{v_0}{2}\left(\frac{\mu/2}{\gamma_G}\exp\left[-\frac{\mu/2}{\gamma_G}Q\right]+\frac{\delta}{\gamma_G}\exp\left[-\frac{\delta}{\gamma_G}Q\right]\right)\exp\left[-\frac{(\beta'-\gamma_G)}{\gamma_G}Q\right]$$
$$<\frac{a_{ss}}{a_0}\frac{1}{a_s}c'(s) - c''(s) + \left(\frac{a_s}{a_0}\right)^2\left(\frac{v_0}{4Q}\right)<0,$$

where the first inequality follows from evaluating the maximum value of the term beginning $(a_s/a_0)^2$ (treating $\mu/(2\gamma_G)\geqslant 0$ as a choice variable) and the second from Condition U. So the maximum is unique. The properties of the schooling effort function are derived by totally differentiating (A.28), to yield:

$$\frac{\partial^2(J(0)_U - c(s))}{\partial^2 s}ds + \frac{\partial^2(J(0)_U - c(s))}{\partial s\partial\gamma_G}d\gamma_G + \frac{\partial^2(J(0)_U - c(s))}{\partial s\partial\mu}d\mu = 0.$$

The effect of an increase in μ. Differentiating (A.28) with respect to μ yields

$$\frac{\beta'^2}{(\beta'-\delta)(\beta'+\frac{1}{2}\mu)^2}\exp[Q]$$
$$-\frac{\mu(\beta'+\frac{\mu}{2})(\delta+\frac{\mu}{2})-\frac{\mu^2}{2}(\delta+\beta'+\mu)-\frac{Q}{2}(\beta'+\frac{\mu}{2})^2(\delta+(\mu/2))^2}{(\beta'+\frac{1}{2}\mu)^2(\delta+\frac{1}{2}\mu)^2}$$
$$\times\exp\left[-\frac{\beta'+\frac{\mu}{2}-\gamma_G}{\gamma_G}Q\right]-\frac{\delta^2}{(\beta'-\delta)(\delta+\frac{1}{2}\mu)^2}\exp\left[-\frac{\beta'-\delta-\gamma_G}{\gamma_G}Q\right]$$
$$>\frac{\beta'^2}{(\beta'-\delta)(\beta'+\frac{\mu}{2})^2}-\frac{\mu(\beta'+\frac{\mu}{2})(\delta+\frac{\mu}{2})-\frac{\mu^2}{2}(\delta+\beta'+\mu)}{(\beta'+\frac{1}{2}\mu)^2(\delta+\frac{1}{2}\mu)^2}$$
$$-\frac{\delta^2}{(\beta'-\delta)(\delta+\frac{\mu}{2})^2}=0,$$

the inequality following since $Q>0$, $\beta'>\gamma_G$, $\beta'-\delta-\gamma_G=\beta+r-\gamma_G>0$, and the coefficient on $\exp[-\frac{\beta'-\delta-\gamma_G}{\gamma_G}Q]$ is strictly positive. The result then follows as $\frac{\partial^2(J(0)_U-c(s))}{\partial^2 s}<0$.

The effect of an increase in γ_G. Differentiating (A.28) with respect to γ_G yields:

$$-\left((\mu^2/2)\exp\left[-\frac{\beta'+\frac{1}{2}\mu-\gamma_G}{\gamma_G}Q\right]+\delta\mu\exp\left[-\frac{\beta'-\delta-\gamma_G}{\gamma_G}Q\right]\right)\frac{Q(a_s/a_0)}{\delta+\frac{1}{2}\mu}\frac{v_0}{2}<0.$$

The result then follows as $\frac{\partial^2(J(0)_U-c(s))}{\partial^2 s}<0$. Similarly, the effect of an increase in K_0 follows. □

Proof of Proposition 4 (Properties of steady-state equilibrium). An equilibrium is characterized by a pair (μ^*,η^*) such that $\eta^*-\eta^{EE}(\mu^*;s,K_0)=0$ and

$\eta^* - \eta^{SS}(\mu^*; m_0) = 0$. The application of standards methods, yields $d\mu^*/ds > 0$, $d\mu^*/dK_0 > 0$ and $d\mu^*/dm_0 > 0$

(i) (*wages*) Totally differentiating w^* with respect to arbitrary λ yields

$$dw^*/d\lambda = \partial w^*/\partial\lambda + (dw^*/d\mu)(d\mu^*/d\lambda).$$

Using Proposition 2, the results follow. The other comparative statics results follow in a similar manner. □

Proof of Proposition 5 (Steady-state equilibrium with endogenous schooling). (i) The proof of existence is virtually the same as in Proposition 3 the SS locus is unchanged and the EE locus adjusts to changes in s.

(ii) From Lemma 4 $\lim_{\mu\to 0}\eta^{EE}(\mu;\cdot) = \eta_0 \in (0, \bar{\eta})$ *with* $\bar{\eta} < \infty$, $\lim_{\mu\to\infty}\eta^{EE}(\mu;\cdot) = \infty$, $\lim_{\gamma_G\to 0}\partial\eta^{EE}/\partial\gamma_G > 0$, if $\mu \in (0, \bar{\mu})$, $\bar{\mu} < \infty$, *and* $\lim_{\gamma_G} \to 0\partial\eta^{EE}/\partial\gamma_G < 0$ otherwise. Furthermore, it follows, from (3.10), that $\lim_{m_0\to 0}\eta^{SS}(\mu;\cdot) = 0$. Hence there exist $\tilde{m}_0$, such that $\mu^* < \bar{\mu}$ whenever $m_0 \in (0, \tilde{m}_0)$ and $\mu^* > \bar{\mu}$ otherwise. It follows that $d\mu^*/d\gamma_G < 0$ if $m_0 \in (0, \tilde{m}_0)$. Totally differentiating w^* with respect to γ_G yields:

$$\lim_{\gamma_G\to 0} dw^*/d\gamma_G = \lim_{\gamma_G\to 0}\{(\partial w^*/\partial\gamma_G) + (\partial w^*/\partial\mu)(d\mu^*/d\gamma_G)\}.$$

Since $\partial w^*/\partial\gamma_G < 0$, $\partial w^*/\partial\mu > 0$ (see Proposition 2) and, as shown above, $d\mu^*/d\gamma_G < 0$ if $m_0 \in (0, \tilde{m}_0)$, the result follows. The other results follow in a similar manner. □

Asset value Equations with re-training:

(a) *The value of employment*

Assume that the firm trains the worker initially at $\hat{q}_0^E(q)$ and then periodically at $\hat{q}^E$. The stationary asset value is,

$$J_{0E} = \frac{\delta}{\gamma_G - \gamma_E}\int_0^{\hat{q}^E} \exp\left[-\frac{\tilde{\beta}+\delta}{\gamma_G - \gamma_E}q'\right] J(q')_U\, dq' + \exp\left[-\frac{\tilde{\beta}+\delta}{\gamma_G - \gamma_E}\hat{q}^E\right] J_{0E}$$

which then yields,

$$J_{0E} = \frac{\frac{\delta}{\gamma_G - \gamma_E}\int_0^{\hat{q}^E} \exp[-\frac{\tilde{\beta}+\delta}{\gamma_G - \gamma_E}q'] J(q')_U\, dq'}{1 - \exp\left[-\frac{\tilde{\beta}+\delta}{\gamma_G - \gamma_E}q'\right]}.$$

Ex ante the worker is first re-trained at $\hat{q}_0^{\mathrm{E}}$. This then gives,

$$J(q)_{\mathrm{E}} = w(q) + \frac{\delta}{\gamma_{\mathrm{G}} - \gamma_{\mathrm{E}}} \int_0^{\hat{q}_0^{\mathrm{E}}} \exp\left[-\frac{\tilde{\beta}+\delta}{\gamma_{\mathrm{G}} - \gamma_{\mathrm{E}}}(q' - q)) \right] J(q')_{\mathrm{U}} dq' + \exp\left[-\frac{\tilde{\beta}+\delta}{\gamma_{\mathrm{G}} - \gamma_{\mathrm{E}}}(\hat{q}_0^{\mathrm{E}}(q) - q) \right] J_{0\mathrm{E}}$$

which then yields,

$$J(q)_{\mathrm{E}} = w(q) + \frac{\delta}{\gamma_{\mathrm{G}} - \gamma_{\mathrm{E}}} \int_q^{\hat{q}_0^{\mathrm{E}}} \exp\left[-\frac{\tilde{\beta}+\delta}{\gamma_{\mathrm{G}} - \gamma_{\mathrm{E}}}(q' - q)) \right] J(q')_{\mathrm{U}}\, dq' + \exp\left(\left[-\frac{\tilde{\beta}+\delta}{\gamma_{\mathrm{G}} - \gamma_{\mathrm{E}}}(\hat{q}_0^{\mathrm{E}}(q) - q) \right] \frac{\frac{\delta}{\gamma_{\mathrm{G}}-\gamma_{\mathrm{E}}} \int_0^{\hat{q}^{\mathrm{E}}} \exp[-\frac{\tilde{\beta}+\delta}{\gamma_{\mathrm{G}}-\gamma_{\mathrm{E}}} \hat{q}^{\mathrm{E}}] J(q)_{\mathrm{U}}\, dq}{1 - \exp[-\frac{\tilde{\beta}+\delta}{\gamma_{\mathrm{G}}-\gamma_{\mathrm{E}}} \hat{q}^{\mathrm{E}}]} \right).$$

When $\gamma_{\mathrm{E}} = \gamma_{\mathrm{G}}$, the worker's q value is invariant on the job. The worker is re-trained at most once. If the worker is not re-trained then $J(q)_{\mathrm{E}} = w(q) + \frac{\delta}{\tilde{\beta}+\delta} J(q)_{\mathrm{U}}$ as he leaves the firm in the same state, q, as he entered. If the worker is re-trained, then $J(q)_{\mathrm{E}} = w(q) + \frac{\delta}{\tilde{\beta}+\delta} J(0)_{\mathrm{U}}$ as he leaves the firm a re-trained worker of the most recent vintage.

(b) *The unemployment valuation*

Assume that the worker re-trains himself upon reaching $\hat{q}^{\mathrm{U}}$. The stationary value of this behavior is then,

$$J_{0\mathrm{U}} = -C(\hat{q}^{\mathrm{U}}) + \frac{\mu}{\gamma_{\mathrm{G}} + \gamma_{\mathrm{U}}} \int_0^{\hat{q}^{\mathrm{U}}} \exp\left[-\frac{\tilde{\beta}+\mu}{\gamma_{\mathrm{G}} + \gamma_{\mathrm{U}}} q' \right] J(q')_{\mathrm{E}}\, dq' + \exp\left[-\frac{\tilde{\beta}+\mu}{\gamma_{\mathrm{G}} + \gamma_{\mathrm{U}}} \hat{q}^{\mathrm{U}} \right] J_{0\mathrm{U}}$$

which then yields,

$$J_{0\mathrm{U}} = \frac{-C(\hat{q}^{\mathrm{U}}) + \frac{\mu}{\gamma_{\mathrm{G}}+\gamma_{\mathrm{U}}} \int_0^{\hat{q}^{\mathrm{U}}} \exp[-\frac{\tilde{\beta}+\mu}{\gamma_{\mathrm{G}}+\gamma_{\mathrm{U}}} q'] J(q')_{\mathrm{E}}\, dq'}{1 - \exp[-\frac{\tilde{\beta}+\mu}{\gamma_{\mathrm{G}}+\gamma_{\mathrm{U}}} \hat{q}^{\mathrm{U}}]}.$$

Ex ante the worker first re-trains himself at, $\hat{q}_0^{\mathrm{U}}$

$$J(q)_{\mathrm{U}} = \frac{\mu}{\gamma_{\mathrm{G}} + \gamma_{\mathrm{U}}} \int_q^{\hat{q}_0^{\mathrm{U}}} \exp\left[-\frac{\tilde{\beta}+\mu}{\gamma_{\mathrm{G}} + \gamma_{\mathrm{U}}}(q' - q) \right] J(q')_{\mathrm{E}} dq' + \exp\left[-\frac{\tilde{\beta}+\mu}{\gamma_{\mathrm{G}} + \gamma_{\mathrm{U}}}(\hat{q}_0^{\mathrm{U}} - q) \right] \{-C(\hat{q}_0^{\mathrm{U}}) + C(\hat{q}^{\mathrm{U}}) + J_{0\mathrm{U}}\},$$

where $-C(\hat{q}_0^{\mathrm{U}}) + C(\hat{q}^{\mathrm{U}})$ is the net re-training cost ($J_{0\mathrm{U}}$ represents a net of re-training cost asset value). □

Proof of Lemma 6 (Re-training on the job). If the worker is initially re-trained, then $\hat{q}^{\mathrm{E}} \to \infty$. Without subsequent re-training the expected value of the worker is: a_0. The value of re-training the worker is: $a_0 - C(q) < a_0$. With $\gamma_{\mathrm{E}} = \gamma_{\mathrm{G}}$, q and hence the value functions are invariant to the length of tenure at the firm. If re-training is pairwise optimal, then with positive discounting, it occurs at the point of hiring. If re-training is not initially optimal, then (given stationarity) it cannot be optimal to re-train the worker at a later date. □

Proof of Proposition 6 (Re-training). Under Condition R(i), differentiating $J(q)_U$ with respect to q gives, for $q < Q^*_E$.

$$\dot{J}(q)_U = -\frac{\mu}{\gamma}(a_0 \exp[-q] - v_0) + \frac{\mu}{2\gamma}[(1-\hat{\mu}) + (1-\hat{\mu}\hat{\delta})]J(q)_U,$$

where $\gamma \equiv \gamma_G + \gamma_E$ and the Nash bargaining solution, $J(q)_E - J(q)_U = a_0 \exp[-q] - w(q) - v_0$, is used. Simple re-arrangement of the boundary values $J(Q^*_E)_E$ and $J(Q^*_E)_U$ yield,

$$J(Q^*_E)_U = \frac{\mu/2}{\tilde{\beta} + \mu/2}\frac{1}{(1-\hat{\delta})}(a_0 \exp[-Q^*_E] - v_0),$$

where Q^*_E solves: $\hat{\delta}J(0)_U = \hat{\delta}J(Q^*_E)_U + C_0 - a_0(1 - \exp[-Q^*_E])$. If $\delta = 0$, then $Q^*_E = -\ln[(a_0 - C_0)/a_0] > 0$ and the boundary value is, $J(Q^*_E)_U = \frac{\mu/2}{\tilde{\beta}+\mu/2}(a_0 \exp[-Q^*_E] - v_0)$, implying the differential equation possesses the solution displayed in (6.12). Upon differentiation, $J(q)_U$ is seen to be decreasing in $q \in [0, Q^*_E)$. In order to ensure that this is an equilibrium, we must ensure that workers do no re-train themselves. We must show that for all $q \in [0, Q^*_E]$, $J(0)_U - C_0 - J(q)_U \leqslant 0$. A sufficient condition is then to show, $J(0)_U - C_0 - J(Q^*_E)_U \leqslant 0$, as $J(q)_U$ is decreasing in q. Simple manipulation gives,

$$\begin{aligned} sgn\{J(0)_U - C_0 - J(Q^*_E)_U\} = &- a_0\gamma\mu + \gamma\mu(a_0 - C_0)\left(1 - \frac{C_0}{a_0}\right)^{\frac{\tilde{\beta}+\mu/2}{\gamma}} \\ &- 2C_0\tilde{\beta}(\gamma + \tilde{\beta} + \mu/2) \\ < &- \gamma\mu(C_0) - 2C_0\tilde{\beta}(\gamma + \tilde{\beta} + \mu/2) < 0 \end{aligned}$$

as claimed. Finally, we show that the firm hires workers for whom, $q \geqslant Q^*_E$, requiring: $\Pi(Q^*_E)_F - v_0 \geqslant 0$. Under the Nash bargain, we have: $J(Q^*_E)_E - J(Q^*_E)_U = \Pi(Q^*_E)_F - v_0$. But,

$$J(Q^*_E)_E = w(Q^*_E) = \frac{1}{2}(a_0 \exp[-Q^*_E] - v_0) + \frac{1}{2}J(Q^*_E)_U,$$

$$J(Q^*_E)_U = \frac{\frac{1}{2}\mu}{\frac{1}{2}\mu + \tilde{\beta}}(a_0 \exp[-Q^*_E] - v_0) = \frac{\frac{1}{2}\mu}{\frac{1}{2}\mu + \tilde{\beta}}(a_0 - C_0)$$

which give,

$$\Pi(Q^*_E)_F - v_0 = \frac{\tilde{\beta}}{\frac{1}{2}\mu + \tilde{\beta}}(a_0 - v_0 - C_0) \geqslant 0,$$

where the inequality follows from Condition R(ii). □

Proof of Proposition 8 (Mandatory retirement). For any worker with history q, recall that $dH(q)/d\tau_E = [\Gamma_E(\tau_E) - \gamma_G]H(q)$. Suppose there does not exist mandatory retirement date $\tau_E(q)_M \in (0, \infty)$. Then, consider such a worker at employment tenure $\tau_E(q) \to \infty$. But Assumption 4 implies that $H(q) \to H_{\min} < H(\gamma_G \tau_U)$. Since re-entry into the labor market is costless for firms under Assumption 5, it is beneficial for a firm matched with a worker of $\tau_E(q) \to \infty$ to do so, thus leading to contradiction. $\square$

References

[1] D. Acemoglu, Training and innovation in an imperfect labour market, Rev. Econom. Stud. 64 (1997) 445–464.

[2] D. Acemoglu, R. Shimer, Holdups and efficiency with search frictions, Int. Econom. Rev. 40 (1999) 827–850.

[3] P. Aghion, P. Howitt, A model of growth through creative destruction, Econometrica 60 (1992) 323–351.

[4] P. Aghion, P. Howitt, Growth and unemployment, Rev. Econom. Stud. 61 (1994) 477–494.

[5] G. Akerlof, Jobs as dam sites, Rev. Econom. Stud. 48 (1981) 37–49.

[6] K. Binmore, A. Rubinstein, A. Wolinsky, The Nash bargaining solution in economic modeling, Rand J. Econom. 17 (1986) 176–188.

[7] K. Burdett, S. Shi, R. Wright, Pricing with Frictions, Working Paper, University of Pennsylvania, Philadelphia, PA, 2000.

[8] V.V. Chari, H. Hopenhayn, Vintage human capital, growth, and the diffusion of new technology, J. Polit. Econom. 99 (1991) 1142–1165.

[9] M. Coles, R. Wright, A dynamic equilibrium model of search, bargaining and money, J. Econom. Theory 78 (1998) 32–54.

[10] M. Coles, A. Masters, Duration Dependent U. I., Payments and Long-Term Unemployment, I.L.R. Working Paper, University of Essex, UK, 2001.

[11] T.F. Cooley, J. Greenwood, M. Yorukoglu, The replacement problem, J. Monet. Econom. 40 (1997) 459–500.

[12] J. De Long, L. Summers, Equipment investment and economic growth, Quart. J. Econom. 106 (1991) 445–502.

[13] P. Diamond, Wage determination and efficiency in search equilibrium, Rev. Econom. Stud. 49 (1982) 217–227.

[14] B.C. Fallick, The Demand for Labor by Expanding Industries, Working Paper, University of California, Los Angeles, CA, 1992.

[15] C. Goldin, R.A. Margo, The great compression: the wage structure in the United States at mid-century, Quart. J. Econom. 107 (1992) 1–34.

[16] G. Grossman, E. Helpman, Quality ladders in the theory of growth, Rev. Econom. Stud. 43 (1991) 43–62.

[17] L. Jacobson, R. LaLonde, D. Sullivan, Earnings losses of displaced workers, Amer. Econom. Rev. 83 (1993) 685–709.

[18] S.C. Kolm, Unequal inequalities. II, J. Econom. Theory 13 (1976) 82–111.

[19] D. Laing, T. Palivos, P. Wang, Learning, matching, and growth, Rev. Econom. Stud. 62 (1995) 115–129.

[20] D. Laing, T. Palivos, P. Wang, Endogenous pricing and product diffusion with goods market frictions, Econom. Theory 19 (2002) 707–736.

[21] R. Layard, S. Nickell, R. Jackman, Unemployment: Macroeconomic Performance and the Labor Market, Oxford University Press, Oxford, UK, 1991.

[22] R.E. Lucas Jr., On the mechanics of economic development, J. Monet. Econom. 22 (1988) 3–42.

[23] J.A. Mincer, Schooling, Experience, and Earnings, Columbia University Press, New York, NY, 1974.
[24] E. Moen, Competitive search equilibrium, J. Polit. Econom. 104 (1997) 724–746.
[25] D.T. Mortensen, Property rights and efficiency in mating, racing and related games, Amer. Econom. Rev. 72 (1982) 968–969.
[26] D.T. Mortensen, C. Pissarides, Job creation and job destruction in the theory of unemployment, Rev. Econom. Stud. 61 (1994) 397–415.
[27] K. Murphy, F. Welch, The structure of wages, Quart. J. Econom. 107 (1992) 285–326.
[28] S. Neuman, A. Weiss, On the effects of schooling vintage on experience-earnings profiles: theory and evidence, Euro. Econom. Rev. 39 (1995) 943–955.
[29] M. Peters, Ex ante price offers in matching games: non-steady state, Econometrica 59 (1991) 1425–1454.
[30] E. Phelps, Inflation Policy and Unemployment Theory, Norton, New York, NY, 1972.
[31] C.A. Pissarides, Efficient job rejection, Econom. J. 94 (1984) S97–S108.
[32] C.A. Pissarides, Loss of skill during unemployment and the persistence of employment shocks, Quart. J. Econom. 107 (1992) 1371–1391.
[33] R. Pollak, Welfare comparisons and equivalence scales, Amer. Econom. Rev. 69 (1979) 216–221.
[34] P.M. Romer, Increasing returns and long-run growth, J. Polit. Econom. 94 (1986) 1002–1037.
[35] C. Ruhm, Are workers permanently scarred by job displacement?, Amer. Econom. Rev. 81 (1991) 319–323.
[36] G. Solon, R. Barsky, J.A. Parker, Measuring the cyclicality of real wages: how important is composition bias?, Quart. J. Econom. 109 (1994) 1–25.
[37] N. Stokey, Human capital, product quality, and growth, Quart. J. Econom. 106 (1991) 587–616.
[38] C.R. Taylor, Time-on-the-market as a signal of quality, Rev. Econom. Stud. 67 (1999) 555–578.