Variable Elasticity of Substitution and Economic Growth: Theory and Evidence[†]

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Abstract

We construct a one-sector growth model where the technology is described by a Variable Elasticity of Substitution (VES) production function. This framework allows the elasticity of factor substitution to interact with the level of economic development. First, we show that the model can exhibit unbounded endogenous growth despite the absence of exogenous technical change and the presence of non-reproducible factors. Second, we provide some empirical estimates of the elasticity of substitution, using a panel of 82 countries over a 28-year period, which admit the possibilities of a VES aggregate production function with an elasticity of substitution that is greater than one and consequently of unbounded endogenous growth.

Key Words: *Elasticity of Substitution, Endogenous Growth, VES Production Functions.*

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1. Introduction

The elasticity of factor substitution plays a crucial role in the theory of economic growth. Among others, it is one of the determinants of the level of economic growth; see, for example, de La Grandville (1989) and Klump and de la Grandville (2000). It affects the speed of convergence towards the balanced growth path; see Klump and Preissler (2000). It can alter the behavior of the savings rate during the transition; see Smetters (2003). It influences the aggregate distribution of income; the seminal work on this topic is Hicks (1932). Finally, it may itself be a source of unbounded growth; see Solow (1956) and Palivos and Karagiannis (2004).

Most papers of economic growth that attempt to provide some quantitative properties of growth models rely on the Cobb-Douglas specification of the production function, which, as it is well known, describes a process with an elasticity of factor substitution equal to one. Recently, several papers in the literature have investigated both theoretically and empirically the role played by the Constant Elasticity of Substitution (CES) production function, which allows the elasticity to take constant values that are either greater or lower than one. Examples include, among others, Klump and de la Grandville (2000), Klump and Preissler (2000), Miyagiwa and Papageorgiou (2003), Duffy *et al.* (2004) and Masanjala and Papageorgiou (2004).

This paper extends this literature a step further by analyzing the role of a variable elasticity of substitution (VES) within a standard Solow-Swan growth model. Whereas the CES production function restricts the elasticity of substitution to be constant along an isoquant, this paper employs a specification, first introduced by Revankar (1971), which allows the elasticity of substitution to interact with the level of economic development. More specifically, a change in the economy's per capita capital affects the elasticity of substitution between capital and labor. This change feeds back into the economy influencing capital accumulation and output. It is shown that the model can exhibit unbounded endogenous growth despite the absence of exogenous technical change and the presence of non-reproducible factors, e.g., labor.

Moreover, the paper uses a panel of 82 countries over a 28-year period to estimate an aggregate production function with variable elasticity of substitution. The estimation results provide first evidence in favor of a VES production function. In addition, the estimated elasticity of substitution in the sample is greater than one, which provides empirical support to the aforementioned theoretical result regarding unbounded endogenous growth.

The remainder of the paper is organized as follows. Section 2 analyzes the properties of Revankar's VES production function. Section 3 introduces this production function in an otherwise standard Solow-Swan model and derives necessary and sufficient conditions for unbounded endogenous growth. Section 4 offers a short review of the previous studies that have estimated VES functions. Section 5 discusses the data, the estimation techniques and the empirical results. Finally, Section 6 concludes the paper.

2. A VES Production Function

2.1. The Revankar VES Production Function

We use standard notation to denote a general production technology as $Y = F(K, L)$, where Y, K, and L stand for output, capital and labor, respectively. Following Revankar (1971), we consider the following specification:[1]

$$Y = AK^{a\nu} \left[L + baK\right]^{(1-a)\nu}. \tag{2.1}$$

We mostly assume that the production function exhibits constant returns to scale, i.e., $\nu = 1$. This production function can be written in intensive form, $y = f(k)$ where $y \equiv Y/L$ and $k \equiv K/L$, as

$$y = Ak^{a} \left[1 + bak\right]^{1-a}. \tag{2.2}$$

It follows that

$$f'(k) = a\frac{y}{k} + a(1-a)b\frac{y}{1+abk}, \tag{2.3}$$

$$f''(k) = Aa(a-1)(1+abk)^{-a-1}k^{-1}. \tag{2.4}$$

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[1] A very similar VES specification was developed by Sato and Hoffman (1968).

Hence, this function satisfies standard properties of a production function, namely $f(k) > 0$, $f'(k) > 0$ and $f''(k) < 0 \; \forall k > 0$, as long as

$$A > 0, \quad 0 < a \leq 1, \quad b > -1 \quad and \quad k^{-1} \geq -b.$$

Note that if $b = 0$ then (2.2) reduces to the Cobb-Douglas case. On the other hand if $a = 1$ then it reduces to the Ak production function.

2.2. Some Properties of the VES

The limiting properties of (2.2) are:

$$\begin{aligned} \lim_{k \to 0} f(k) &= 0, \quad \lim_{k \to \infty} f(k) = \infty \quad if \quad b > 0 \\ \lim_{k \to -b^{-1}} f(k) &= A(-b)^{-a}(1-a)^{1-a} > 0 \quad if \quad b < 0 \end{aligned} \tag{2.5}$$

Furthermore, it follows from (2.3) that

$$\begin{aligned} \lim_{k \to 0} f'(k) &= \infty, \quad \lim_{k \to \infty} f'(k) = A(ba)^{1-a} > 0 \quad if \quad b > 0, \\ \lim_{k \to -b^{-1}} f'(k) &= A\left[-b(1-a)\right]^{1-a} > 0 \quad if \quad b < 0. \end{aligned} \tag{2.6}$$

Thus, if $b > 0$ then one of the two Inada conditions is violated; namely, the marginal product of capital is strictly bounded from below, which is equivalent to labor not being an essential factor of production, i.e., if $b > 0$, then $\lim_{L \to 0} F(K, L) = A(ba)^{1-a} > 0$.

The labor share, s_L, implied by (2.2) is:

$$\begin{aligned} s_L &= \frac{1-a}{1+bak}, \quad \text{where} \quad \lim_{k \to 0} s_L = 1 - a, \\ \lim_{k \to \infty} s_L &= 0 \;\; if \;\; b > 0 \quad \text{and} \quad \lim_{k \to -b^{-1}} s_L = 1 \quad \text{if } b < 0. \end{aligned} \tag{2.7}$$

On the other hand, the properties of the capital share, s_K, follow easily since $s_K = 1 - s_L = \frac{a+bak}{1+bak}$.

For this production function, the elasticity of substitution between capital and labor $\sigma(x) = -\frac{f'(x)}{xf(x)} \frac{f(x)-xf'(x)}{f''(x)} > 0$ is

$$\sigma(k) = 1 + bk > 0. \tag{2.8}$$

Hence, $\sigma \gtreqless 1$ if $b \gtreqless 0$. Thus, the elasticity of substitution varies with the level of per capita capital, an index of economic development. Furthermore, σ plays an important role in the development process. To see why, note that (2.1) can be written as:

$$Y = AK^a L^{1-a} \left[1 + ba\frac{K}{L} \right]^{1-a},$$

or, using (2.8),

$$Y = AK^a L^{1-a} \left[1 - a + a\sigma(k) \right]^{1-a}. \tag{2.9}$$

Hence, the production process can be decomposed into a Cobb-Douglas part, $AK^a L^{1-a}$, and a part that depends on the (variable) elasticity of substitution, $\left[1 - a + a\sigma(k) \right]^{1-a}$. Once again, if $b = 0$ then $\sigma = 1$ and

$$Y = AK^a L^{1-a},$$

which is the Cobb-Douglas production function. In intensive form (2.1) is written as

$$y = Ak^a \left[1 - a + a\sigma(k) \right]^{1-a}. \tag{2.10}$$

Some of the properties of the VES are also shared by the CES. Exceptions include the elasticity of substitution which for the CES production function is constant along an isoquant, while for the VES considered here it is constant only along a ray through the origin (see equation 2.8). Also, factor shares behave slight differently, since for the CES $\lim_{k \to 0} s_L = 1$ if $\sigma > 1$ and $\lim_{k \to 0} s_L = 0$ if $\sigma < 1$.

3. VES in the Solow-Swan Growth Model

Next we introduce this VES specification in a standard Solow-Swan growth model (Solow 1956). The accumulation equation is

$$\frac{\dot{k}}{k} = s\frac{f(k)}{k} - n, \tag{3.1}$$

where s denotes the savings rate and n stands for the population growth rate. Using (2.10), we have

$$\frac{f(k)}{k} = Ak^{a-1} \left[1 - a + a\sigma(k) \right]^{1-a},$$

$$\lim_{k \to x} \frac{f(k)}{k} = \lim_{k \to x} f'(k), \qquad x = 0, \infty, b^{-1}$$

where $\lim_{k\to x} f'(k)$ is given by (2.6). Also,

$$\frac{\partial(f(k)/k)}{\partial k} = -A(1-a)k^{a-2}\left[1+bak\right]^{-a} < 0.$$

Upon substitution, equation (3.1) becomes

$$\frac{\dot{k}}{k} = sAk^{a-1}\left[1-a+a\sigma(k)\right]^{1-a} - n. \tag{3.2}$$

If $b > 0$ and hence $\sigma > 1$, the properties of the growth rate of per capita capital $\dot{k}/k$ are

$$\lim_{k\to 0}\frac{\dot{k}}{k} = \infty \qquad \text{and} \qquad \lim_{k\to\infty}\frac{\dot{k}}{k} = sA\left(ba\right)^{1-a} - n.$$

Thus, if $sA\left(ba\right)^{1-a} > n$, then the model exhibits unbounded endogenous growth; that is, there exists an asymptotic balanced growth path with positive per capita growth. This result is consistent with the findings of Jones and Manuelli (1990, 1997), who show that unbounded growth can occur despite the presence of non-reproducible factors, i.e., labor, and the absence of exogenous technical progress, as long as the marginal product of capital is strictly bounded from below. It is also consistent with that results in Palivos and Karagiannis (2004), which shows that an elasticity of substitution that becomes asymptotically (as k grows) greater than one is necessary and sufficient for the existence of a lower bound on the marginal product of capital. The following graph illustrates the possibility of unbounded growth.

This possibility arises also with a CES production function as long as $\sigma > 1$. However, in this model the process is more explicit, since as it can be seen from (3.2) an increase in k affects the growth rate $\dot{k}/k$ through two channels. The first is through sAk^{a-1} for any given σ. This term is decreasing in k (the Cobb-Douglas part). The second is the change in σ, which is linear in k. So an increase in output raises σ, which raises output even further.

If $sA\left(ba\right)^{1-a} < n$, then the growth rate will eventually become zero. The economy will reach a steady state, which is given as the solution to the following equation (see Figure 2)

$$sA(k^*)^{a-1}[1-a+a\sigma(k^*)]^{1-a} = n. \tag{3.3}$$

Consider next the case where $-1 < b < 0$ and $0 < k \leq -(1/b)$. In this case, if $sA\left[-b(1-a)\right]^{1-a} < n$, then there is again a unique steady state, given by (3.3) (see Figure 3). On the other hand, if $-1 < b < 0$ and $sA\left[-b(1-a)\right]^{1-a} > n$, then the system will reach a corner solution, where $k = -1/b$ (Figure 4).

4. Empirical Considerations of VES

The previous empirical studies using a VES production function (see Table 4.1) can be divided into two groups depending on whether they have used time-series or cross-section data.[2] The former group includes the studies of Sato and Hoffman (1968), Lovell (1968), Revankar (1971b), Lovell (1973b), Roskamp (1977) and Bairam (1989, 1990). Sato and Hoffman (1968), using data from the private non-farm sector of the U.S. and Japan, concluded that "the overall impression is that the VES is more realistic than the CES," without however providing a formal statistical test. Revankar (1971b), on the other hand, using data for the private non-farm sector of the U.S., formally rejected the Cobb-Douglas form in favor of the VES, while Lovell (1973b) could not reject the CES specification in favor of the VES for the U.S. manufacturing sector as a whole. Nevertheless, Lovell (1968) rejected both the Cobb-Douglas and the CES specifications in favor of the VES for 16 two-digit U.S. manufacturing industries. Moreover, Bairam (1989, 1990) rejected the Cobb-Douglas in favor of the VES specification for the Japanese and Soviet economies. Roskamp (1977), using data for manufacturing in Germany, provided estimates of the elasticity of substitution for 38 industries using both the CES and the VES, without formally testing for the most appropriate specification. With the exception of Roskamp (1977), in 7 out of 38 industries, and of Bairam (1989), these time-series studies estimated the elasticity of substitution to be less than one.

The remaining studies reported in Table 4.1 fall in the group of cross-section studies. Lu and Fletcher (1968) formally rejected the CES in favor of the VES specification in 7 to 9 (depending on various definitions of capital and labor inputs) out of the 17 two-digit manufacturing sectors included in their analysis. Similarly, Revankar (1971a) rejected the Cobb-Douglas in favor of the VES specification in 5 out of 12 two-digit U.S. manufacturing sectors. Lovell (1973a) rejected the Cobb-Douglas and the CES in favor of the VES specification in 3 out of 17 two-digit U.S. manufacturing sectors. Kazi (1980) rejected in most cases the CES in favor of the VES specification. Furthermore, Diwan (1970), using even more micro data for individual U.S. manufacturing firms, rejected both the Cobb-Douglas and the CES specifications in favor of the VES. A similar result was reached by Meyer and Kadiyala (1974), who used agricultural experimental data. Finally, Tsang and Yeung (1976) and Zellner and Ryu (1998) provided estimates of both the CES and the VES

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[2] Our (incomplete) review covers only production function that are linearly homogeneous.

for respectively the food and kindred products and transportation equipment industries in the U.S., but they did not formally tested for the more appropriate specification. With the exception of Lu and Fletcher (1968) and of Kazi (1980), these cross-section studies gave estimates of the elasticity of substitution that were less than one.

5. Estimation of a VES Production Function

Whether or not the aggregate production technology is VES is an empirical question. We now turn our attention toward this estimation exercise. Our estimation of a VES specification for the aggregate production involves data on 82 countries for 28 years (1960-1987).[3] We consider nonlinear least squares (NLLS) regressions to obtain our parameter estimates. We begin by briefly describing the data used in our estimation.

5.1. The data

All of the raw data that we use are obtained from the World Bank's STARS database. In particular, GDP and the aggregate physical capital stock are converted into constant, end of period 1987 $U.S. The database also provides us with data on the number of individuals in the workforce between the ages of 15-64, as well as data on the mean years of schooling of members of the workforce. In addition to considering raw (unadjusted) labor, L, as an input in our VES specification, we also examined whether adjusting labor input for human capital accumulation affects our results. Here we follow Tallman and Wang (1994) and adopt a simple proxy for human capital adjusted labor input. First, we define the stock of human capital in country i at time t, H_{it}, as $H_{it} = E_{it}$, where E_{it} denotes the mean years of schooling of the workforce (workers between the ages of 15-64 as in the measure of L) in country i at time t. The mean school years of education, E, is defined as the sum of the average number of years of primary, secondary and post-secondary education. Then we define *human capital adjusted labor supply* as $HL_{it} = H_{it} \times L_{it} = E_{it} \times L_{it}$. In estimating the VES specification for aggregate production, we will use both L and HL as measures of labor input. Further details concerning the construction of these data are provided in Duffy and Papageorgiou (2000) and mean values of all relevant variables appear in the appendix.

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[3] These data are from Duffy and Papageorgiou (2000).

5.2. Estimation equation

Taking logs of both sides of (2.1) and assuming that technology grows exogenously at rate λ (i.e., $A = A_0 e^{\lambda t}$) yields our estimation equations:

$$\begin{aligned} \log Y_{it} &= \log A_0 + \lambda t + a\nu \log K_{it} + \\ &\quad + (1-a)\nu \log\left[L_{it} + baK_{it}\right] + \varepsilon_{it}, \end{aligned} \tag{5.1}$$

$$\begin{aligned} \log Y_{it} &= \log A_0 + \lambda t + a \log K_{it} + \\ &\quad + (1-a) \log\left[L_{it} + baK_{it}\right] + \varepsilon_{it}, \end{aligned} \tag{5.2}$$

where A_0 is initial technology, i is country index, t is time and ε is a random error. Note that in our estimations we consider both cases of non-constant ($\nu \neq 1$) and constant ($\nu = 1$) returns to scale. We estimated equations (5.1-5.2) by nonlinear least squares (NLLS) for the entire panel of 2,296 observations using our data on real GDP, physical capital and either raw labor supply, L, or human capital adjusted labor supply, HL, in place of L. The initial parameter choices for all of the NLLS estimation results reported in Table 1 were based on estimates we obtained from a preliminary OLS regression of $\log Y_{it}$ on a constant, $\log K_{it}$ and $\log L_{it}$ or $\log HL_{it}$. We also considered other initial parameter choices and obtained similar NLLS estimates.

The second column of Table 1 presents estimates for the unrestricted ($\nu \neq 1$) VES production function given by equation (5.1). All of the estimated coefficients are significantly different from zero at the 1 percent level and economically plausible, regardless of whether L or HL is used for labor input. Consistent with other studies using similar data, the time trend coefficient is negative and significant ($\lambda = -0.012, -0.014$) indicating that for the 82 countries of our sample, the log of real GDP has, on average, declined slightly over the period 1960-1987. The coefficients for a are 0.66781 and 0.70473 (and highly significant) for the models using raw and adjusted labor, respectively.

The key finding regarding our testable hypothesis is that the sign for the coefficient estimate b is found to be positive for both types of labor input and significant, thus providing first evidence of a VES aggregate production function. In particular, the estimated coefficient for b is 0.00050 for the unrestricted model using raw labor and 0.00141 for the same model using skilled labor. These estimates may at first seem too small but closer observation of their potential impact on the elasticity of substitution (i.e., $\sigma = 1 + bk$)

Table 5.1: Nonlinear Regression Estimates

Labor (L)	Unrestricted ($\nu \neq 1$) NLLS	Restricted ($\nu = 1$) NLLS
a	0.66781***	0.67283***
	(0.06176)	(0.03770)
b	0.00050***	0.00046***
	(0.00018)	(0.00015)
λ	-0.01170***	-0.01177***
	(0.00093)	(0.00091)
A_0	24.753***	24.822***
	(1.8109)	(1.8028)
ν	0.99779***	—
	(0.00501)	—
-ln L	837.58	837.68
Adj. Labor (HL)		
a	0.70473***	0.73468***
	(0.06358)	(0.03775)
b	0.00141*	0.00070**
	(0.00083)	(0.00031)
λ	-0.01401***	-0.01549***
	(0.00098)	(0.00090)
A_0	29.336***	31.517***
	(1.9191)	(1.9013)
ν	0.97126***	—
	(0.00488)	—
-ln L	955.96	974.50
Obs.	2,296	2,296

Note: Standard errors are given in parentheses. *** Significantly different from 0 at the 1% level. ** Significantly different from 0 at the 5% level. * Significantly different from 0 at the 10% level.

suggests otherwise. Further, our results imply that the elasticity of substitution between capital and labor, σ, is in general greater than one. Given that the coefficient estimates for b are found to be different from zero, we can reject the Cobb-Douglas specification, for our 28 year and 82 country sample, over the more general VES specification.

Finally, for the unrestricted models the coefficient estimate for ν is shown to be very close to unity ($\nu = 0.99779$). Thus, the constant-returns-to-scale (CRTS) restriction seems reasonable for the case where raw labor is used as input. Interestingly, the same is not true for the model using adjusted labor (HL) as the labor input since $\nu = 0.97126$ which is consistent with mild diminishing-returns-to-scale (DRTS). However, since the theory supposes that there are constant returns to scale in production, we also estimate the "restricted version" of the model above, using equation (5.2).

The results for the restricted ($\nu = 1$) VES production function are presented in the third column of Table 1. We see that while the magnitude of the NLLS estimates for all parameters in the restricted model differ slightly from those obtained using the unrestricted model, the signs and statistical significance of the coefficient estimates are largely unchanged by comparison. Once again the key parameter b is positive in sign and very significant when we use raw labor ($b = 0.00046$). However, when we restrict the model and use adjusted labor the coefficient estimate increases considerably ($b = 0.00070$) than that in the unrestricted model and becomes significant only at the 5 percent level. This result is not surprising because restricting the model with HL to obey CRTS results in compromising the accuracy of the coefficient estimate b.

Another interesting finding from our NLLS estimation concerns the implied country-specific labor and capital shares (s_L and s_K, respectively). In the special Cobb-Douglas case, the parameter b is equal to zero (see equation 5.2) and the terms $1 - a$ and a are readily interpreted as the labor and capital shares of output. However, under the VES specification, the labor share is given by $s_L = \frac{1-a}{1+bak}$ and the capital share by $s_K = \frac{a+bak}{1+bak}$. Therefore both shares depend on the values of K, L, a and most importantly b. Since our estimated coefficients for b are positive and significantly different from zero, it follows that factor shares vary with a country's capital-labor ratio. This finding is important in light of Kaldor's (1961) "stylized facts" about the shares of income accruing to capital and labor being relatively constant over time and countries. This view has been first challenged by the pioneer paper of Solow (1958) and remains today an open research question (see, for example, Gollin (2002) who finds that labor's share of national income across 31 countries is relatively constant). Our results certainly suggest that capital shares can vary considerably across countries and increase with the capital-labor ratio and therefore with economic development.

To summarize, the main finding from our nonlinear estimation exercises is that the coefficient estimates of b are found to be positive and significantly different from zero, implying a variable elasticity of substitution between capital and labor that is in general greater than unity. Of course, this is in contrast to the aggregate Cobb-Douglas production specification assumed by most theoretical and empirical studies.

6. Conclusions and Extensions

We have analyzed a one-sector growth model with a variable elasticity of substitution production function. We have shown that the model can exhibit unbounded endogenous growth despite the absence of exogenous technical change and the presence of non-reproducible factors, such as labor. Second, we have used a panel of 82 countries over a 28-year period to estimate an aggregate production function. Our empirical estimates of the elasticity of substitution support the possibility of unbounded endogenous growth.

In future work we plan to examine the robustness of our baseline OLS results when we correct for the fixed effects and endogeneity problems usually cited in the literature. Thus far, the aggregate input-output production relationship we have estimated using NLLS does not allow for the presence of *fixed effects* across countries. A "fixed-effects" specification would allow us to capture country-specific characteristics, e.g., geography, political factors or culture, that might affect aggregate output. Admitting the possibility of fixed effects implies that the error term in (5.1-5.2) can be written as $\varepsilon_{it} = \eta_i + \upsilon_{it}$, where η_i captures time-invariant fixed factors in country i. Given this specification, first differencing (5.1-5.2) gets rid of the fixed effect component in the error term, yielding the nonlinear equations:

$$\begin{aligned} \log\left(\frac{Y_{it}}{Y_{i,t-1}}\right) &= \lambda + a\nu\log\left(\frac{K_{it}}{K_{i,t-1}}\right) + \\ &\quad +(1-a)\nu\log\left[\frac{L_{it}+baK_{it}}{L_{i,t-1}+baK_{i,t-1}}\right] + \upsilon_{it} - \upsilon_{i,t-1} \end{aligned} \tag{6.1}$$

$$\begin{aligned}\log\left(\frac{Y_{it}}{Y_{i,t-1}}\right) &= \lambda + a\log\left(\frac{K_{it}}{K_{i,t-1}}\right) + \\ &+(1-a)\log\left[\frac{L_{it}+baK_{it}}{L_{i,t-1}+baK_{i,t-1}}\right] + \upsilon_{it} - \upsilon_{i,t-1}. \qquad (6.2)\end{aligned}$$

While it is straightforward to estimate (6.1-6.2) using NLLS, the first-difference specification leads to another difficulty in that the lagged error term, $\upsilon_{i,t-1}$, is likely to be correlated with time t values of the explanatory variables, K_{it} and L_{it}. More generally, the capital accumulation equation used to construct the capital stock values (see Appendix A for details) implies that K_{it} will *always* depend on such lagged error terms.[4] We plan to use a generalized method of moments (GMM) approach to estimate the parameters in (6.1-6.2), which is a more general estimation method than nonlinear two stage estimation in that the GMM approach allows for the possibility of both autocorrelation and heteroskedasticity in the disturbance term, $\upsilon_{it} - \upsilon_{i,t-1}$. Thus, it seems appropriate in the present context.

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[4] The first paper that examined cross-country growth regressions adjusting for both the fixed-effects problem as well as for the endogeneity problem is Caselli *et al.* (1996). For further discussion on these issues the reader is referred to their paper.

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Appendix

Country	Code	GDP (bill. US$)	Capital (bill. US$)	Labor (mill. age 15–64)	Education (avg yrs of edu.)
Algeria	DZA	38.7	142	7.87	2.51
Argentina	ARG	90	250	16	6.38
Australia	AUS	136	426	8.51	6.55
Austria	AUT	83.7	240	4.74	8.7
Bangladesh	BGD	11.3	22.4	38.3	2.56
Brazil	BRA	162	420	59	3.13
Belgium	BEL	104	274	6.24	7.87
Bolivia	BOL	3.62	13.3	2.59	4.29
Cameroon	CMR	6.4	9.75	4.03	1.68
Canada	CAN	260	600	14	8.98
Chile	CHL	13.8	35.5	5.9	6.06
China	CHN	103	309	513	3.36
Colombia	COL	21.5	48.2	13	3.54
Côte d'Ivoire	CIV	6.65	14	3.35	0.93
Costa Rica	CRI	2.87	10.9	1.04	6.14
Cyprus	CYP	1.91	6.15	0.38	6.91
Denmark	DEN	74.5	199	3.21	8.36
Ecuador	ECU	6.52	20.1	3.60	4.22
Egypt	EGY	17.2	25.5	19.9	3.59
El Salvador	SLV	3.71	6.19	1.96	3.54
Ethiopia	ETH	3.93	3.86	17.2	0.24
Finland	FIN	58.6	199	3.11	8.2
France	FRA	629	1620	33	8.01
Germany	DEU	831	2420	42	8.43
Ghana	GHA	4.27	8.77	4.97	2.98
Greece	GRC	31.5	82.1	5.90	7.76
Guatemala	GTM	5.08	10.1	3.04	2.72
Haiti	HTI	1.78	2.18	2.66	1.9
Honduras	HND	2.58	5.03	1.55	3.23
Iceland	ICE	3.08	7.96	0.129	7.58
Indonesia	IND	39	59.3	72.3	2.91
India	IND	155	365	343	2.37
Iran	IRN	109	183	17	2.02
Iraq	IRQ	49	71.6	5.62	2.33
Ireland	IRL	19.5	47.8	1.84	14.55
Israel	ISR	21.9	59.8	1.95	4.69
Italy	ITA	511	1480	36	6.96
Jamaica	JAM	2.71	13.3	1.04	6.89
Japan	JPN	1400	3600	74	10.67

Country	Code	GDP (bill. US$)	Capital (bill. US$)	Labor (mill. age 15–64)	Education (avg yrs of edu.)
Jordan	JOR	3.06	5.44	1.21	3.11
Kenya	KEN	4.36	19.2	6.53	2.48
Korea, Rep.	KOR	51.6	87.7	19.1	5.12
Madagascar	MDG	2.33	3.83	4.05	2.4
Malawi	MWI	0.77	2.03	2.68	3.34
Malaysia	MYS	16.3	34.5	6.54	4.32
Mali	MLI	1.36	3.34	3.04	0.49
Mauritius	MUS	1.02	3.63	0.5	5.41
Mexico	MEX	89.2	206	31	4.36
Morocco	MAR	11.1	25.1	8.7	1.33
Mozambique	MOZ	1.59	5.91	5.67	1.65
Myanmar (Burma)	MMR	6.95	12	16.7	1.68
Netherlands	NLD	159	483	8.59	8.1
New Zealand	NZL	26.8	77.5	1.8	7.06
Nigeria	NGA	22.4	68.8	37.6	1.34
Norway	NOR	51.9	204	2.48	8.87
Pakistan	PAK	16.7	31.8	36.3	1.49
Panama	PAN	3.14	7.04	0.92	5.66
Paraguay	PRY	2.12	4.12	1.41	5.42
Peru	PER	18.6	52.8	8.0	4.79
Philippines	PHI	23.5	49.5	22.5	6.14
Portugal	PRT	23.9	75.6	5.98	4.44
Rwanda	RWA	1.33	1.09	2.16	2.09
Senegal	SEN	3.32	6.55	2.61	0.98
Sierra Leone	SLE	0.44	0.83	1.59	1.21
Singapore	SGP	9.26	24.5	1.39	4.68
Spain	ESP	201	494	22	6.01
Sri Lanka	LKA	3.95	7.5	7.59	5.15
Sudan	SDN	12.4	13.8	8.44	0.88
Sweden	SWE	120	320	5.25	9.12
Switzerland	CHE	134	374	4.09	6.62
Tanzania	TZA	2.39	7.44	7.84	1.23
Thailand	THA	23.3	48.6	21.7	4.61
Tunisia	TUN	5.36	16.6	3.01	3.0
Turkey	TUR	37.1	93.2	21.9	3.11
Uganda	UGA	5.33	9.31	5.31	2.1
United Kingdom	GRB	510	1220	36	9.66
United States	USA	3100	8300	135	10.91
Uruguay	URY	5.96	18.4	1.77	6.07
Venezuela	VEN	37.2	116	6.71	4.28
Zaire	ZAR	6.2	8.1	11.9	2.57
Zambia	ZMB	1.76	11.9	2.42	2.55
Zimbabwe	ZWE	3.62	12.6	2.94	3.54

Figure 1. $b > 0$ and $sA(ba)^{1-a} > n$

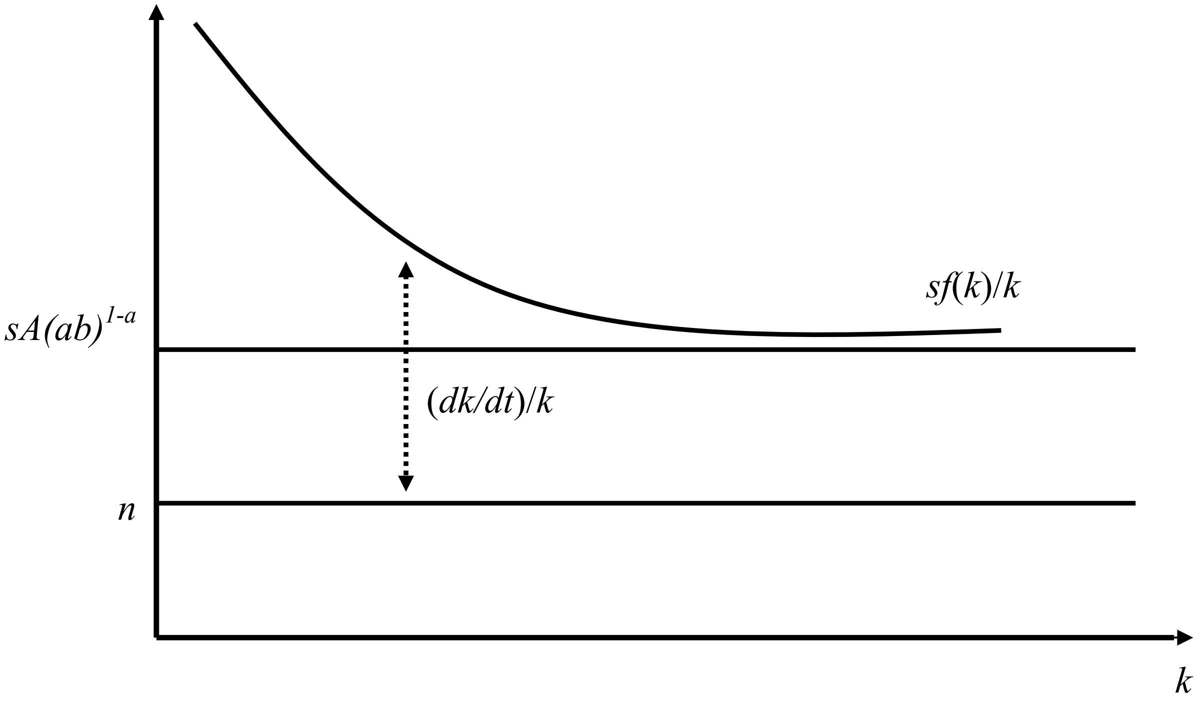

Figure 2. $b > 0$ and $sA(ba)^{1-a} < n$

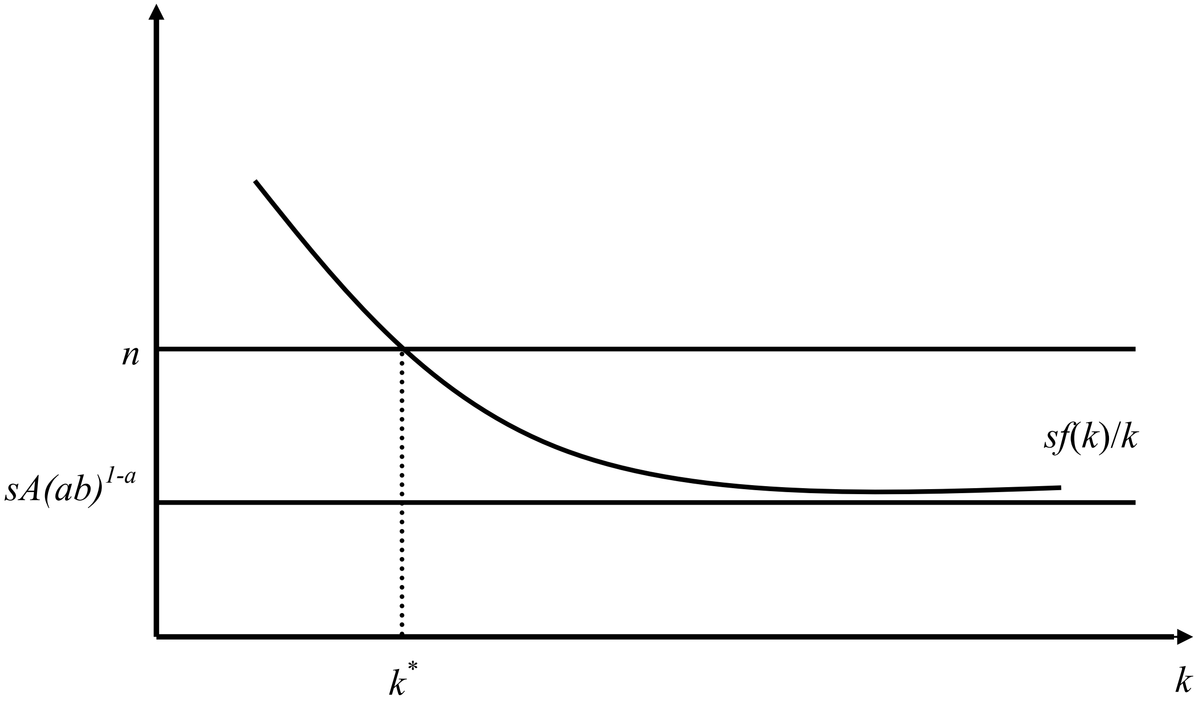

Figure 3. $b < 0$ **and** $sA(-b(1-a))^{1-a} < n$

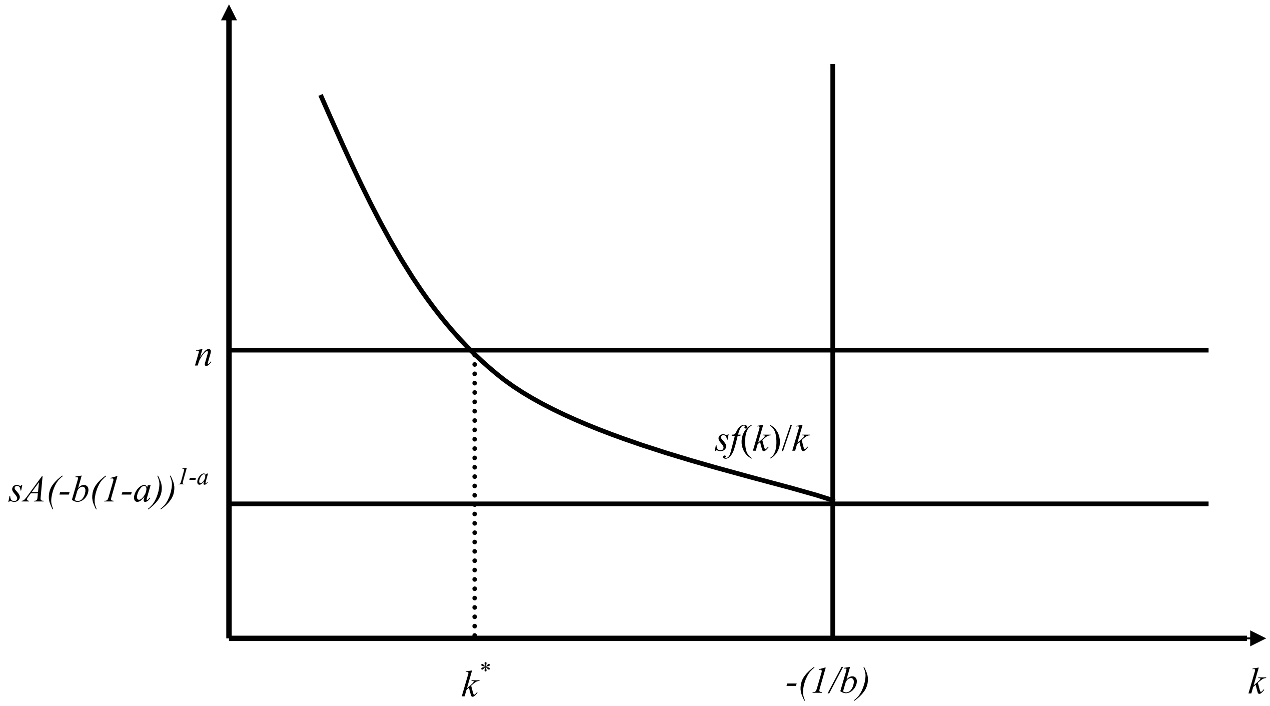

Figure 4. $b < 0$ **and** $sA(-b(1-a))^{1-a} > n$

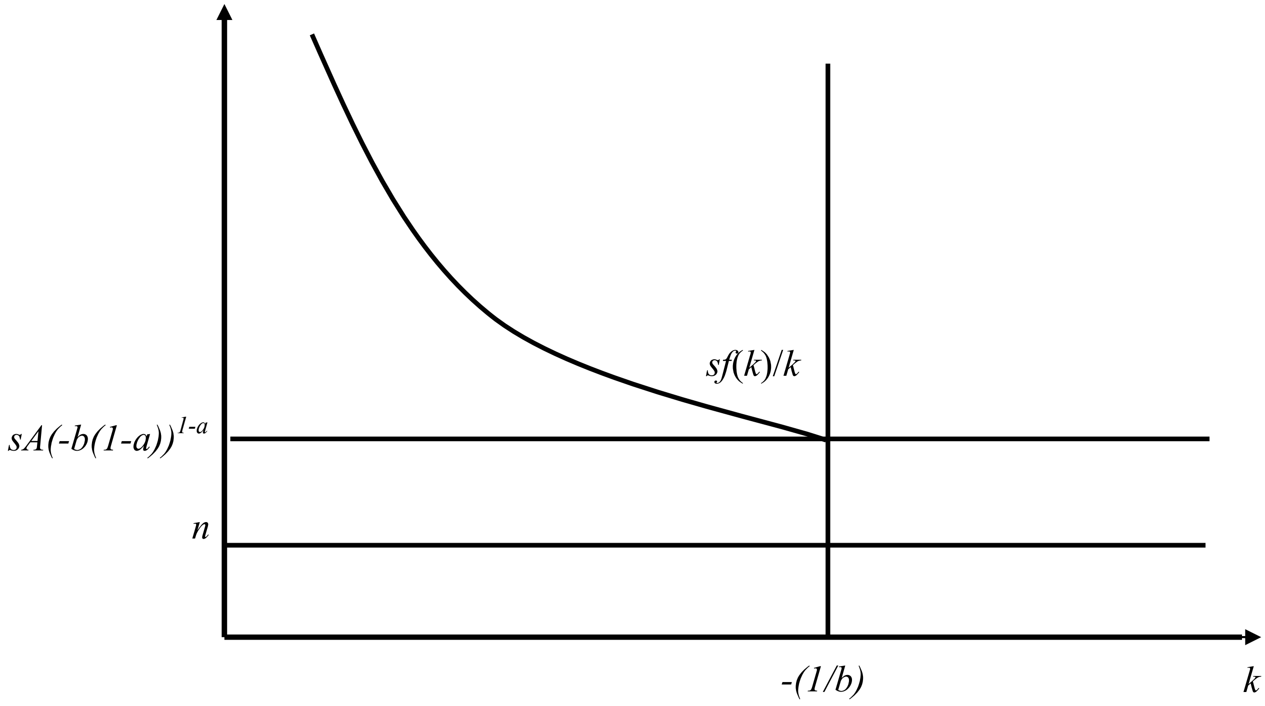

Table 4.1: Previous Empirical Considerations of VES

Study	Country	Period	Sector
Lu and Fletcher (1968)	U.S.	1957	Two-digit manufacturing
Sato and Hoffman (1968)	U.S.	1909-60	Private non-farm sector
	Japan	1930-60	Private non-farm sector
Lovell (1968)	U.S.	1949-63	Two-digit manufacturing
Diwan (1970)	U.S.	1955-57	Manufacturing firms
Revankar (1971a)	U.S.	1957	Two-digit manufacturing
Revankar (1971b)	U.S.	1929-53	Private non-farm sector
Lovell (1973a)	U.S.	1958	Two-digit manufacturing
Lovell (1973b)	U.S.	1947-68	Manufacturing
Meyer and Kadiyala (1974)	U.S.		Agriculture
Tsang and Yeung (1976)	U.S.	1957	Food & kindred products
Roskamp (1977)	Germany	1950-60	Manufacturing
Kazi (1980)	India	1973-75	Two- & Three-digit manufacturing
Bairam (1989)	Japan	1878-1939	Economy
Bairam (1990)	U.S.S.R.	1950-75	Economy & manufacturing
Zellner and Ryu (1998)	U.S.	1957	Transportation equipment