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Optimal population size and endogenous growth

Theodore Palivos *

Department of Economics, Louisiana State University, Baton Rouge, LA 70803, USA

Chong K. Yip

Georgia State University, Atlanta, GA, USA

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Abstract

We compare the population and output growth rates implied by the welfare functions of Bentham and Mill. Contrary to previous findings, we show that, under conditions that ensure an optimum, the Benthamite criterion leads to a smaller population size and higher economic growth.

1. Introduction

Many applications in economics require the selection of an objective function which enables the comparison of allocations involving different population sizes. The two most commonly used criteria are the Benthamite and the Millian welfare functions, also known as classical and average utilitarianism, respectively. The former maximizes total utility of the society and thus represents individuals, while the latter maximizes average utility and so represents generations.

Edgeworth (1925) was the first to conjecture that the Benthamite principle leads to a larger population size and a lower standard of living. This is one of the main objections to classical utilitarianism [see Dasgupta (1984)] and thus per capita utility maximization is often used to justify limits in population. In two recent articles, Nerlove et al. (1982, 1985) confirm Edgeworth's conjecture. Specifically, they show that, in a finite horizon setting in which there is no production and parents care about the number and the utility of their children, the optimal population growth rate is larger for the Benthamite than for the Millian welfare function. They also reach the same conclusion in an infinite horizon setting making steady-state comparisons.

The purpose of this paper is to examine Edgeworth's conjecture in an endogenous growth framework in which there are interactions between output and population growth rates. It is shown that, under conditions that ensure an optimum, the Benthamite criterion leads to smaller population and higher output growth rates than the Millian.

* Corresponding author.

2. The model

Consider a continuous-time optimal growth model in which a representative agent (generation) seeks to maximize his/her lifetime utility. The instantaneous utility function depends on per capita consumption, $c(t)$, and the population growth rate, $n(t)$. [1] In the study of a growing economy, such as the one examined here, the appropriate analogue to a steady state is that of a balanced growth path (defined below). To ensure the existence of such a path, however, it is necessary to assume that the instantaneous utility function takes the constant intertemporal elasticity form $u(c(t), n(t)) = [c(t)^{\alpha} n(t)^{1-\alpha}]^{\sigma}/\sigma$, where $0<\alpha<1$ and $\sigma<1$ [see King et al. (1988)]. The (constant) intertemporal elasticity of substitution in consumption is then equal to $1/(1-\alpha\sigma)$. Furthermore, in the absence of exogenous technical progress, the existence of a balanced growth equilibrium requires also that the production technology exhibits constant returns to scale with respect to per capita capital. Following Rebelo (1991), we assume $y(t) = Ak(t)$, where A denotes the (constant) marginal product of capital and $y(t)$ denotes per capita output. Given initial values of N and k, the representative agent (generation) then faces the following problem:

$$\max W = \int_0^{\infty} [c(t)^{\alpha} n(t)^{1-\alpha}]^{\sigma}/(\sigma) N(t)^{1-\epsilon}\, \mathrm{e}^{-\rho t}\, \mathrm{d}t\,, \qquad (\mathcal{P})$$

subject to

$$\dot{k}(t) = Ak(t) - c(t) - n(t)k(t)\,, \qquad (1)$$

$$\dot{N}(t) = n(t)N(t)\,, \qquad (2)$$

$$c(t) \geq 0\,, \qquad n(t) \geq 0\,, \qquad k(t) \geq 0\,, \quad \text{for } t \geq 0\,,$$

$$k(0) = k_0 > 0\,, \qquad N(0) = N_0 > 0\,,$$

where $\rho > 0$, $N > 0$, and $t \geq 0$ denote the rate of time preference, total population size, and a time index, respectively. We restrict ϵ to take values in the closed unit interval. More specifically, if ϵ is equal to zero (one), then the optimality criterion in ($\mathcal{P}$) becomes the Benthamite (Millian) social welfare function. It can also be shown that a necessary condition for $\mathrm{d}W/\mathrm{d}n$ to attain the value zero and thus for an optimum to exist is $\sigma < 0$, which we henceforth assume. [2]

3. Balanced growth analysis

Necessary and sufficient conditions for the existence of an optimal path are the optimality conditions with respect to c and n:

$$\alpha c(t)^{\alpha\sigma-1} n(t)^{(1-\alpha)\sigma} N(t)^{1-\epsilon} = \lambda_1(t)\,, \qquad (3)$$

$$(1-\alpha)c(t)^{\alpha\sigma} n(t)^{(1-\alpha)\sigma-1} N(t)^{1-\epsilon} = \lambda_1(t)k(t) - \lambda_2(t)N(t); \qquad (4)$$

the Euler equations:

[1] Notice that there is a one-to-one correspondence between the population growth rate and the number of children.
[2] The proof is presented in an appendix available upon request.

$$\dot{\lambda}_1(t) = \rho\lambda_1(t) + n(t)\lambda_1(t) - A\lambda_1(t) , \tag{5}$$

$$\dot{\lambda}_2(t) = [\rho - n(t)]\lambda_2(t) - (1-\epsilon)c(t)^{\alpha\sigma}n(t)^{(1-\alpha)\sigma}N(t)^{-\epsilon}/\sigma; \tag{6}$$

the transversality conditions:

$$\lim_{t\to\infty} \mathrm{e}^{-\rho t}\lambda_1(t)k(t) = 0 \quad \text{and} \quad \lim_{t\to\infty} \mathrm{e}^{-\rho t}\lambda_2(t)N(t) = 0 ,$$

and the constraints (1) and (2), where λ_1 and λ_2 denote the costate variables associated with them.

Next, we turn to the balanced growth equilibrium path. By definition, along this path per capita consumption, per capita capital, and the population size grow at a constant rate. First, notice that Eq. (2) implies a common growth rate (θ) for c and k. Moreover, by taking logarithms of both sides of (3) and then differentiating with respect to time, one obtains

$$(\alpha\sigma - 1)\theta + (1-\epsilon)n = \dot{\lambda}_1/\lambda_1 . \tag{7}$$

Or, by using (5),

$$(1-\alpha\sigma)\theta = A - \rho - \epsilon n , \tag{8}$$

i.e. the return to consumption, $(1-\alpha\sigma)\theta + \rho - (1-\epsilon)n$, should equal the net return to investment, $A - n$. Next divide (4) by (3) to obtain

$$\lambda_2 N/\lambda_1 c = k/c - (1-\alpha)/\alpha n , \tag{9}$$

which in turn implies

$$\dot{\lambda}_2/\lambda_2 + n = \dot{\lambda}_1/\lambda_1 + \theta . \tag{10}$$

Furthermore, substituting consecutively (3), (9), and (1) in (6) yields

$$\dot{\lambda}_2/\lambda_2 = \rho - n + \frac{n(1-\epsilon)(A-\theta-n)}{\sigma[(1-\alpha)(A-\theta)-n]} . \tag{11}$$

Finally, using (7) and (11) to eliminate $\dot{\lambda}_1/\lambda_1$ and $\dot{\lambda}_2/\lambda_2$ in (10), one obtains

$$\sigma(1-\alpha)(\theta - A) = (1-\epsilon-\sigma)n . \tag{12}$$

Solution of the simultaneous equation system consisting of (8) and (12) provides the following analytical expressions for the population growth rate,

$$n = \frac{\sigma(1-\alpha)(\alpha\sigma A - \rho)}{(1-\sigma)(1-\alpha\sigma-\epsilon)} , \tag{13}$$

and the (common) growth rate of capital, consumption, and output,

$$\theta = \frac{A-\rho}{1-\alpha\sigma} - \frac{\epsilon\sigma(1-\alpha)(\alpha\sigma A-\rho)}{(1-\alpha\sigma)(1-\sigma)(1-\alpha\sigma-\epsilon)} . \tag{14}$$

We are now ready to explore the consequences of the social welfare functions of Bentham and Mill. Differentiating (13) with respect to ϵ we get

$$\mathrm{d}n/\mathrm{d}\epsilon = \frac{\sigma(1-\alpha)(\alpha\sigma A - \rho)}{(1-\sigma)(1-\alpha\sigma-\epsilon)^2}\,. \tag{15}$$

Notice that for the integral (lifetime utility) in ($\mathcal{P}$) to be bounded and thus for a maximum to exist, the following inequality must hold: $\alpha\sigma\theta + (1-\epsilon)n < \rho$, or by using (14), $\alpha\sigma A - \rho < 0$. Given $\sigma < 0$, we can conclude from (15) that $\mathrm{d}n/\mathrm{d}\epsilon > 0$. Finally, differentiating (14) with respect to ϵ and comparing it with (9), we get $\mathrm{d}\theta/\mathrm{d}\epsilon = -\mathrm{d}n/\mathrm{d}\epsilon < 0$. Thus, the Benthamite criterion leads to a smaller population size and higher economic growth.

Notice that $\sigma < 0$ plays a very crucial role in obtaining this result. Theoretically, this condition is necessary for the existence of an optimum. Empirically, the intertemporal elasticity of substitution has been found to be far below unity [see, for example, Hall (1988)]. Since the intertemporal elasticity of substitution is $1/(1-\alpha\sigma)$, this also implies that $\sigma < 0$.

Thus far we have restricted our analysis to the balanced growth path. However, one can show that, despite the presence of two stock variables (k and N), there are no transitional dynamics in this model, [3] just as in other one-sector endogenous growth models [e.g. Rebelo (1991)]. Thus, consumption, capital, output, and population grow at the constant rates given by (13) and (14) all the time. We summarize these findings in the following proposition.

Proposition *Suppose the representative generation faces the problem given by* ($\mathcal{P}$). *If an optimum exists, then along any competitive equilibrium path the Benthamite criterion* ($\epsilon = 1$) *leads to a smaller population size and a higher rate of economic growth than the Millian* ($\epsilon = 0$).

[3] The proof is presented in an appendix available upon request.

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